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Properties of Matter



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Chapter (1)

Dimensions and Units

Fundamental concepts of the Physics start from this chapter. Basically the terms & concepts which are illustrated in this topic will be used in so many ways because all Physical quantities have units. It is must to measure all Physical quantities so that we can use them. In this chapter we will have an over view of different units of different Physical quantities. We will learn the dimension and dependence of the unit of any Physical quantity on fundamental quantities or unit.

Three “Fundamental” Physical Quantities

1. Length
2. Mass
3. Time

These three form a “fundamental set” in the sense that they can be used to derive the dimensions of (some) other physical quantities.

By the dimensions of a physical quantity, we mean, “what kind of physical quantity is it (length, mass, time, some combination of these three, etc.).”

Systems of Standard Units

There are three systems of units:

1. SI (“mks system”)

- Length → meter (m)
- Mass → kilogram (kg)
- Time → second (s)

2. cgs system

- Length → centimeter (cm)
- Mass → gram (g)
- Time → second (s)

3. British system

- Length → foot (ft)
- Mass → slug*
- Time → second (s)

Abbreviations for “three fundamental” dimensions:

- **Length = L**
- **Mass = M**
- **Time = T**

These Abbreviations are called **the dimensional equation** of the physical quantity.

The following EXAMPLES show the dimensional equation of some known physical quantities:

1. The area

In general, the area is obtained by multiplying two lengths together. This means that the dimensional equation of area L^2 .

Its units: (MKS system) $\rightarrow \text{m}^2$

(cgs system) $\rightarrow \text{cm}^2$

2. The density

The density of a substance is defined as the mass of a substance divided by its volume, In other words,

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

But since the dimension of the mass is M that of the volume is L^3 , we can easily see that the dimensional equation of density is $\mathbf{ML}^{-3}$.

Its units: (MKS system) $\rightarrow \text{Kg.m}^{-3}$

(cgs system) $\rightarrow \text{gm.cm}^{-3}$

3. The velocity

Velocity is defined as the rate of change of distance with respect to time. i.e.

$$\text{Velocity} = \frac{\text{distance}}{\text{time}}$$

Hence the dimensional equation of velocity is $\mathbf{LT}^{-1}$.

Its units: (MKS system) $\rightarrow \text{m.s}^{-1}$, (cgs system) $\rightarrow \text{cm.s}^{-1}$

4. The acceleration

Acceleration is defined as the rate of change of velocity with respect to time.

$$\text{Acceleration} = \frac{\text{Velocity}}{\text{time}}$$

Hence the dimensional equation of acceleration is $\mathbf{LT^{-2}}$.

Its units: (MKS system) $\rightarrow \text{m.s}^{-2}$

(cgs system) $\rightarrow \text{cm.s}^{-2}$

5. The work

Work is defined as the force multiplied by the distance through which it acts. This means that:

$$\text{Work} = \text{force} \times \text{distance}.$$

But force is equal to acceleration multiplied by mass M.

Also the dimensional equation of acceleration is $\mathbf{LT^{-2}}$.

Hence the dimensional equation of work is given by:

$$\mathbf{M.LT^{-2}.L = ML^2T^{-2}}$$

Its units: (MKS system) $\rightarrow \text{kg.m}^2.\text{s}^{-2}$

(cgs system) $\rightarrow \text{gm.cm}^2.\text{s}^{-2}$

The following Table gives the dimensions of some physical quantities which are usually dealt with in studying Physics or Mechanics.

Physical quantity	Definition	Dimensions	Cgs units
Area	length $\times$ length	L^2	cm^2
Volume	(length) $\times$ (length) $\times$ (length)	L^3	cm^3
Density	mass/volume	ML^{-3}	$gm.cm^{-3}$
Velocity	distance/time	LT^{-1}	$cm.sec^{-1}$
Acceleration	velocity/time	LT^{-2}	$cm.sec^{-2}$
Force	mass $\times$ acceleration	MLT^{-2}	$gm.cm.sec^{-2}$
Work	force $\times$ distance	ML^2T^{-2}	$gm.cm^2.sec^{-2}$
Power	work/time	ML^2T^{-3}	$gm.cm^2.sec^{-3}$
Pressure	force/area	$ML^{-1}T^{-2}$	$gm.cm^{-1}.sec^{-2}$
Momentum	mass $\times$ velocity	MLT^{-1}	$gm.cm.sec^{-1}$
Modulus of elasticity	stress/strain or, force/area	$ML^{-1}T^{-2}$	$gm.cm^{-1}.sec^{-2}$
Moment of force	force $\times$ distance	ML^2T^{-2}	$gm.cm^2.sec^{-2}$

Notes:

1) There are quantities which have no dimensions (or their dimensional equation is equal to 1). They called non-dimensional quantities such as angles and ratios.

2) In the above table one can easily see that some physical quantities have the same dimensions, such as work and moment of force, both have dimensions $\mathbf{ML^2T^{-2}}$.

Also both pressure and modulus of elasticity have the same dimensions, $\mathbf{ML^{-1}T^{-2}}$.

Applications of Dimensional Equation

Dimensional equations are found to be useful in studying physics and mechanics.

The following examples may illustrate the usefulness of such dimensional equations:

A. Testing the correctness of physical laws:

The correctness of any physical relation or law can be tested by obtaining the dimensional equation of both sides. If the dimensions of both sides agree, the physical relation under test is considered to be correct. The following examples may illustrate the above fact:

(1) The relation which gives the period of oscillation of a simple pendulum:

The period of complete oscillation is given by the following relation:

$$T = 2\pi \sqrt{\frac{L}{g}}$$

where,

T: is the time of one complete oscillation (sec.), π is 22/7,
L: is the length of the pendulum (cm), g is the earth gravity acceleration ($\text{cm}.\text{sec}^{-2}$).

- (a) The dimension of the left hand side of the relation is **T**.
- (b) the dimensional equation of the right-hand side of the relation is:

$$\left(\frac{L}{L \cdot T^{-2}} \right)^{1/2} = T$$

Hence, the dimensions of L.H.S. = the dimensions of R.H.S = T.

This proves that, the above relation who governs the period of oscillation of the simple pendulum is **correct**.

(2) The velocity of a transverse wave:

The velocity of a transverse vibration of a string is given by the relation:

$$V = \sqrt{\frac{T}{m}}$$

where:

V: is the velocity of the transverse wave (cm. sec^{-1}), T: the tension of the string (dynes), m: the mass per unit length of the string.

Deriving the dimensional equation for each side of the relation, we get:

(a) For the left hand side: $\mathbf{LT}^{-1}$

(b) For the right hand side, we get:

Tension is force, whose dimensional equation can be written as:

$$MLT^{-2}$$

Also the mass per unit length has a dimensional equation equal to ML^{-1} .

Hence, the dimensional equation of the right hand side is:

$$\left(\frac{MLT^{-2}}{ML^{-1}} \right)^{1/2} = \mathbf{LT}^{-1}$$

So, the dimensions of L.H.S. = the dimensions of R.H.S.

This proves that the relation, who governs the velocity of transverse vibrations given by a string, is **correct**.

B. Deciding the nature of any constant contained in a physical quantity:

Usually physical quantities contain a constant whose nature may not be known. Such a constant may be a pure number (with no dimensions), or may be itself a physical quantity which has dimensions.

In the case, we apply the theory of the dimensional equations to discover the nature of such constant.

The following examples illustrate the significance of such application:

(1) The nature of the constant G included in Newton's general law of gravitation. This law can be written as:

$$F = G \cdot \frac{m_1 m_2}{d^2}$$

where,

F: the force of attraction between the two masses (dynes), m_1 and m_2 , the two masses (gm), d , the distance which lies between the centers of the two masses (cm), G : the gravitational constant.

The constant G can be written as:

$$G = \frac{F \cdot d^2}{m_1 m_2}$$

So, the dimensions of G equal to the dimensions of R.H.S.

The dimensions of R.H.S can be written as:

$$\frac{MLT^{-2} \cdot L^2}{M \cdot M}$$

This means that:

The dimensions of $G = M^{-1} \cdot L^3 \cdot T^{-2}$

and its units are: (MKS system) $\rightarrow \text{kg}^{-1} \cdot \text{m}^3 \cdot \text{s}^{-2}$

(cgs system) $\rightarrow \text{gm}^{-1} \cdot \text{cm}^3 \cdot \text{s}^{-2}$

Hence, the gravitational constant **G is not a pure number.**

It is a physical quantity and **has dimensions**.

(2) The constant K included in the law which governs the relation between the periods of oscillation of the simple pendulum:

$$T = k \sqrt{\frac{l}{g}}$$

where, T is the time of one complete oscillation (sec.), L: is the length of the pendulum (cm), g is the earth gravity acceleration (cm.sec⁻²).

Find the nature of constant k?????

The constant k is given by:

$$k = \frac{T}{(l/g)^{1/2}}$$

the dimensions of k equal to the dimensions of R.H.S which can be written as:

$$R.H.S. = \frac{T}{(L.LT^{-2})^{\frac{1}{2}}}$$

$$= \frac{T}{(L^2.T^{-2})^{1/2}} = 1$$

This means that the constant K is a **pure number**

C. The determination of dimensions of an unknown physical quantity:

This can be illustrated by the following example:

By experiment, Coulomb could discover the following relation:

$$F = \frac{1}{k} \frac{Q_1 Q_2}{d^2}$$

Where, F, the force of attraction or repulsion between the two point-charges (dynes), K, the dielectric constant of the medium which lies between the two charges, Q_1 , the first electric charge, Q_2 , the second electric charge, d, the distance between the two charges (cm).

Obtaining the dimensions of both sides of the above equation, we get:

$$[MLT^{-2}] = \frac{1}{k} L^{-2} [Q]^2$$

Hence, we can write:

$$[Q] = M^{1/2} L^{3/2} T^{-2} K^{1/2}$$

This gives the dimensions of the electrostatic charge.

D. The discovery of a new physical law:

The application of dimensional equations may lead to the discovery of a new physical relation.

D: gm. Cm^{-3} , the density of its material, and E, Young's modulus of the material it is made of.

Obtaining the dimensional equation of each side of the above relation, we may write:

- i) The dimension of the left hand-side is T.
- ii) The dimensions of the right hand-side is given by:

$$[L]^a \cdot [ML^{-3}]^b \cdot [ML^{-1}T^{-2}]^c$$

Hence, the dimensional equation of the above relation can be written as follows:

$$[T] = [L]^a \cdot [ML^{-3}]^b \cdot [ML^{-1}T^{-2}]^c$$

Equation the powers of M, L, and T in each one of the two sides of the dimensional equation, we get:

- 1) Comparing the powers of T in both sides we get:

$$1 = -2c \text{ or } c = -1/2$$

- 2) Comparing the powers of M in both sides we get:

$$0 = b + c \text{ or } b = 1/2$$

- 3) Comparing the powers of L in both sides we get:

$$0 = a - 3b - c$$

$$\text{or } a = 1$$

Putting the numerical value obtained for each of a, b, and c, in the original equation which was assumed, we get:

$$T = k l D^{1/2} E^{-1/2}$$

$$T = k \sqrt{\frac{D}{E}}$$

Which gives the exact law which governs the relation between the period of oscillation of the tuning fork and both its density and elasticity.

Problems

1. Check dimensionally the following equations:

i. $v^2 = v_o^2 + 2at$, where, v : velocity and a : acceleration.

ii. $Fx = 1/2mv^2 - 1/2mv_o^2$, where F : force, x : distance, v_o : initial velocity, v : final velocity

2. Find the dimensions of the following physical quantities:

Kinetic energy – Potential energy – Power – Relative density.

3. Name the physical quantities represented by the following dimensions:

$$LT^{-1} - ML^2T^{-2} - ML^{-1}T^{-2}$$

4. Mention two pairs of physical quantities which have the same dimensions.

5. The centripetal force (F) acting on a particle moving uniformly in a circle depends upon its mass (m), velocity (v) and radius of circle (r). Derive the

expression for centripetal force using method of dimensions.

Chapter (2)

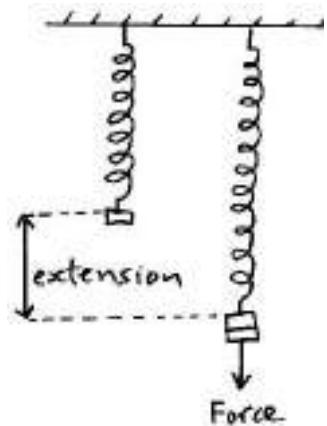
Elasticity

Elasticity is the property of solid materials to return to their original shape and size after the forces deforming them have been removed. Recall Hooke's law — first stated formally by Robert Hooke.

Hooke's Law — Stress and Strain

Hooke's Law

The extension of an elastic object (like a spring) is directly proportional to the force applied, provided the limit of proportionality is not exceeded.



$$F = -k \Delta x$$

where, F : is the applied force.

Δx : is the extension.

K : a constant of proportionality.

This is Hooke's law for a spring — a simple object that's essentially one-dimensional. Hooke's law can be generalized to...

Stress is proportional to strain

Stress refers to the cause of the change (a force applied to a surface)

Strain refers to a change in some spatial dimension (length, angle, or volume) compared to its original value.

The coefficient (proportionality constant) that relates a particular type of stress to the strain that results is called an **elastic modulus**. Elastic moduli are properties of materials, not objects. There are three basic types of stress and three associated moduli.

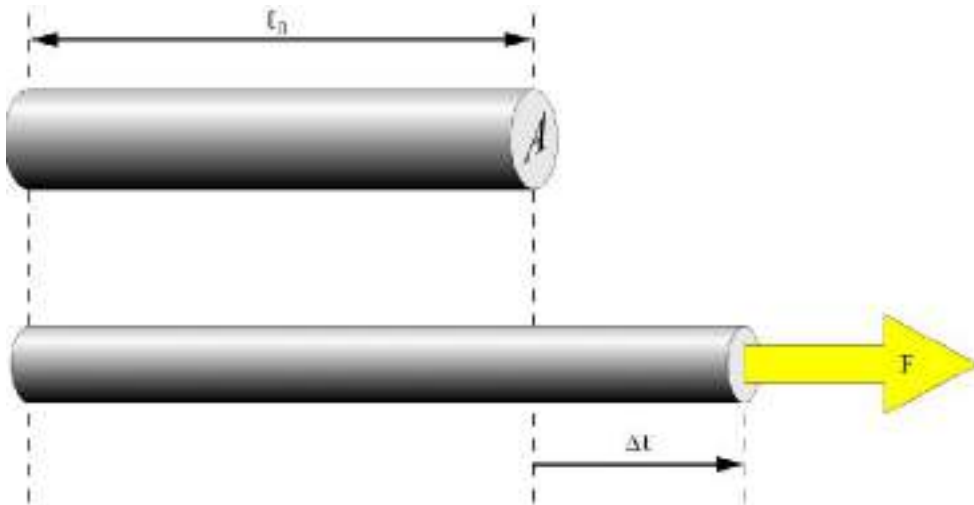
1. Young Modulus (E or Y).
2. Bulk Modulus (K or B).
3. Shear Modulus (G or S).

Elastic moduli			
modulus (symbols)	stress (symbol)	strain (symbol)	configuration change
young's (E or Y)	normal to opposite faces (σ)	length $\epsilon = \Delta l / l_0$	longer and thinner or shorter and fatter
shear (G or S)	tangential to opposite faces (τ)	tangent $\gamma = \Delta x / y$	rectangles become parallelograms
bulk (K or B)	normal to all faces, pressure (P)	volume $\theta = \Delta V / V_0$	volume changes but shape does not

1. Young's Modulus

For the description of the elastic properties of linear objects like wires, rods, columns which are either stretched or compressed, a convenient parameter is the ratio of the stress to the strain, a parameter called the Young's modulus of the material. Young's modulus can be used to predict the elongation or compression of an object as long as the stress is less than the yield strength of the material.

Let's imagine a rod of length ℓ_0 and cross sectional area A being stretched by a force F to a new length $\ell_0 + \Delta\ell$.



Tensile stress (σ) is the outward normal force per unit cross-sectional area A

$$\text{Tensile stress} = \frac{\text{Force}}{\text{Area}} = \frac{F}{A}$$

Its dimensions = $ML^{-1}T^{-2}$

Its units: (MKS system) $\rightarrow \text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-2} \equiv \text{N} \cdot \text{m}^{-2}$

(cgs system) $\rightarrow \text{gm} \cdot \text{cm}^{-1} \cdot \text{s}^{-2} \equiv \text{dyne} \cdot \text{cm}^{-2}$

Tensile strain (ϵ) is the ratio between the change in length to original length of the rod.

$$\text{Tensile strain} = \frac{\text{change in length}}{\text{original length}} = \frac{\Delta l}{l_0}$$

Tensile strain dimensions = 1

Tensile strain units: has no unit.

The proportionality constant that relates these two quantities together is the ratio of tensile stress to tensile strain — **Young's modulus**.

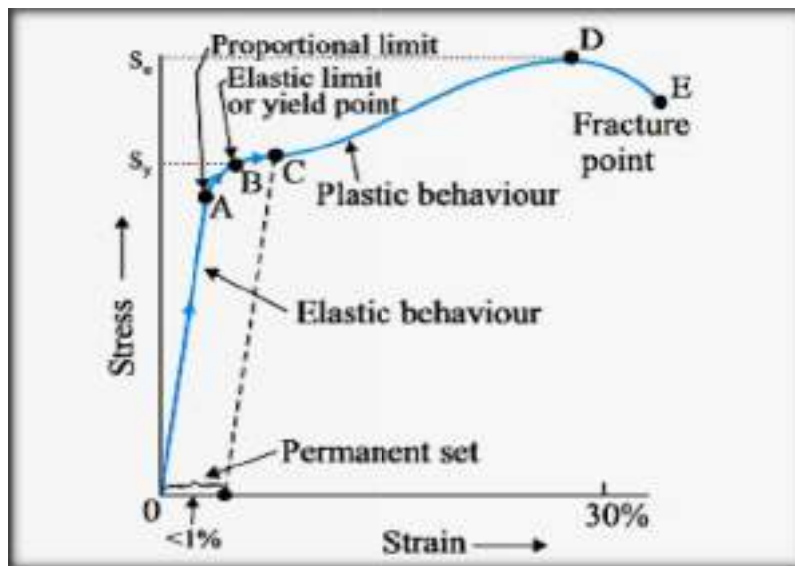
$$\text{Young's modulus (Y)} = \frac{\text{Tensile stress}}{\text{Tensile strain}} = \frac{F/A}{\Delta l/l_0}$$

Its dimensions = the dimensions of stress = $ML^{-1}T^{-2}$

Its units: (MKS system) $\rightarrow kg \cdot m^{-1} \cdot s^{-2} \equiv N \cdot m^{-2}$

(cgs system) $\rightarrow gm \cdot cm^{-1} \cdot s^{-2} \equiv dyne \cdot cm^{-2}$

➤ Stress-Strain Curve



The relation between the stress and the strain for a given material under tensile stress can be found tensile properties, a test cylinder or a wire is experimentally. In a standard test of tensile properties, a test cylinder or a wire is stretched by an applied force. The fractional change in length (the strain) and the applied force needed to cause the strain are recorded. A graph is plotted between the stress (which is equal in magnitude to the applied force per unit area) and the strain produced. The stress-strain curves vary from material to material. These curves help us to understand how a

given material deforms with increasing loads. From the graph, we can see that:

- ❖ In the region between O to A, the curve is linear and Hooke's law is obeyed (when the applied stress is removed, the material return to its original shape).
- ❖ In the region from A to B, stress and strain are not proportional.
- ❖ The point B in the curve is known as **yield point** (also known as **elastic limit**) and the corresponding stress is known as **yield strength (S_y)** of the material.
- ❖ The portion of the curve between B and D shows, if the load is increased further, the stress developed exceeds the yield strength and strain increases rapidly even for a small change in the stress.
- ❖ When the load is removed say at some point C between B and D, the body does not regain its original dimension. In this case, even when the stress is zero, the strain is not zero. The material is said to have a **permanent set**. The deformation is said to be **plastic deformation**.

- ❖ The point D on the graph is the ultimate **tensile strength** (S_u) of the material beyond this point, additional strain is produced even by a reduced applied force and **fracture occurs at point E**. If the ultimate strength and fracture points D and E are close, the material is said to be brittle. If they are far apart, the material is said to be elastic.

Example 1

A structural steel rod has a radius of 10 mm and a length of 1.0 m. A 100 k N force stretches it along its length. Calculate (a) stress, (b) elongation, and (c) strain on the rod. Young's modulus, of structural steel is $2.0 \times 10^{11} \text{ N m}^{-2}$.

Solution

We assume that the rod is held by a clamp at one end, and the force F is applied at the other end, parallel to the length of the rod. The other end, parallel to the length of the rod then the stress on the rod is given by

$$\text{Stress} = \frac{F}{A} = \frac{F}{\pi r^2}$$

$$\text{Stress} = \frac{100 \times 10^3 \text{ N}}{3.14 \times 10^2 \text{ m}^2} = 3.18 \times 10^8 \text{ N.m}^2$$

$$\Delta L = \frac{(F/A)L}{Y} = \frac{3.18 \times 10^8 \text{ N.m}^2 \times 1 \text{ m}}{2 \times 10^{11} \text{ N.m}^2}$$

$$\Delta L = 1.59 \times 10^{-3} \text{ m} = 1.59 \text{ mm}$$

The strain is given by

$$\text{Strain} = \frac{\Delta L}{L} = \frac{(1.59 \times 10^{-3} \text{ m})}{(1 \text{ m})} = 1.59 \times 10^{-3}$$

Example 2

A steel wire 1.00 m long with a diameter $d = 1.00 \text{ mm}$ has a 10.0-kg mass hung from it. (a) How much will the wire stretch? (b) What is the stress on the wire? (c) What is the strain? ($Y = 21 \times 10^{10} \text{ N/m}^2$)

Solution

a. The cross-sectional area of the wire is given by

$$A = \frac{\pi d^2}{4} = \frac{\pi (1.00 \times 10^{-3})^2}{4} = 7.85 \times 10^{-7} \text{ m}^2$$

The force stretching the wire is the weight of the 10.0-kg mass, that is,

$$F = mg = (10.0\text{kg})(9.80\text{m/s}^2) = 98.0\text{ N}$$

The elongation of the wire is given by,

$$\begin{aligned}\Delta L &= \frac{FL_o}{AY} \\ &= \frac{(98.0\text{N})(1.00\text{m})}{(7.85 \times 10^{-7}\text{m}^2)(21.0 \times 10^{10}\text{N/m}^2)} \\ &= 0.594 \times 10^{-3}\text{m} = 0.594\text{mm}\end{aligned}$$

b. The stress acting on the wire is

$$\frac{F}{A} = \frac{98.0\text{ N}}{7.85 \times 10^{-7}\text{m}^2} = 1.25 \times 10^8\text{ N/m}^2$$

c. The strain of the wire is

$$\frac{\Delta L}{L} = \frac{0.594 \times 10^{-3}\text{m}}{1.00\text{ m}} = 0.594 \times 10^{-3}$$

Example3

A copper wire of length 2.2 m and a steel wire of length 1.6 m, both of diameter 3.0 mm, are connected end to end. When stretched by a load, the net elongation is found to be 0.70 mm. Obtain the load applied.

Solution

The copper and steel wires are under a tensile stress because they have the same tension (equal to the load W) and the same area of cross-section A . From Eq. (3. we have

$$\text{Stress} = \text{strain} \times \text{Young's modulus}$$

Therefore

$$\frac{W}{A} = Y_c \times \left(\frac{\Delta L_c}{L_c} \right) = Y_s \times \left(\frac{\Delta L_s}{L_s} \right)$$

Where the subscripts c and s refer to copper and stainless steel respectively

Or,

$$\Delta L_c / \Delta L_s = (Y_s / Y_c) \cdot (L_c / L_s)$$

Given

$$L_c = 2.2 \text{ m}, L_s = 1.6 \text{ m},$$

$$Y_c = 1.1 \times 10^{11} \text{ N.m}^{-2}, Y_s = 2.0 \times 10^{11} \text{ N.m}^{-2}$$

$$\Delta L_c / \Delta L_s = (2.0 \times 10^{11} / 1.1 \times 10^{11}) \times (2.2 / 1.6) = 2.5.$$

The total elongation is given to be

$$\Delta L_c + \Delta L_s = 7.0 \times 10^{-4} \text{ m}$$

Solving the above equations,

$$\Delta L_c = 5.0 \times 10^{-4} \text{ m},$$

$$\Delta L_s = 2.0 \times 10^{-4} \text{ m}.$$

Therefore

$$W = A \times Y_c \times \Delta L_c / L$$

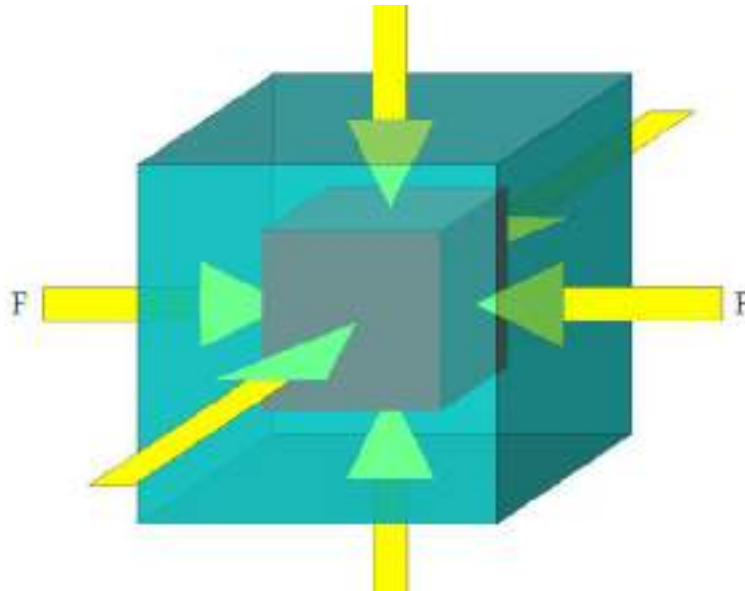
$$W = \pi (1.5 \times 10^{-3})^2 \times [(5 \times 10^{-4} \times 1.1 \times 10^{11}) / 2.2]$$

$$= 1.8 \times 10^2 \text{ N}$$

2. Bulk Modulus

If a uniform force is exerted on all sides of an object, as in the figure, such as a block under water, each

side of the block is compressed (the volume of the object compress without changing its shape). Thus, the entire volume of the block decreases.



The **compression (bulk stress)** is defined as:

$$\text{Bulk stress} = \frac{\text{Force}}{\text{Area}} = \frac{F}{A}$$

where F is the magnitude of the normal force acting on the cross-sectional area A of the block. The **bulk strain** is measured by the change in volume per unit volume, that is,

$$\text{Bulk strain} = \frac{\text{change in volume}}{\text{original volume}} = \frac{\Delta V}{V_0}$$

Since the stress is directly proportional to the strain, by Hooke's law, we have

$$\frac{F}{A} \propto \frac{\Delta V}{V_0}$$

To obtain equality, we introduce a constant of proportionality B, called the **bulk modulus**,

$$\frac{F}{A} = -B \frac{\Delta V}{V_0}$$

The minus sign is introduced because an increase in the stress (F/A) causes a decrease in the volume, leaving ΔV negative. The bulk modulus is a measure of how difficult it is to compress a substance.

$$\text{Bulk modulus (B)} = \frac{\text{Bulk stress}}{\text{Bulk strain}} = \frac{F/A}{\Delta V/V_0}$$

In dealing with liquids and gases it is convenient to deal with the pressure exerted by the liquid or gas. The

pressure is defined as the force that is acting over a unit area of the body, that is,

$$P = \frac{F}{A}$$

For the case of volume elasticity, the stress F/A , acting on the body by the fluid, can be replaced by the pressure of the fluid itself. Thus, Hooke's law for bulk modulus can also be written as

$$P = -\frac{B\Delta V}{V_0}$$

The bulk modulus of a gas which obeys Boyle's law at constant temperature is given by the pressure of the gas, for we may state Boyle's law as

$$PV = \text{constant}$$

and, taking differentials of this equation, we find

$$PdV + V dP = 0,$$

And, solving for P , we find

$$P = -\frac{dP}{dV/V}$$

If we replace the differentials by small increments so that the differential dP is replaced by ΔP and the differential dV is replaced by ΔV , we have

$$P = - \frac{dP}{dV/V} = B$$

From the above analysis it can be seen that, if the pressure of a gas is changed by any amount while the temperature is kept constant, the bulk modulus will vary and, at any stage of the process, will be equal to the pressure of the gas at that stage. This is the reason that tires or basketballs inflated to high pressures seem hard, or difficult to deform, while the same object inflated to low pressure is easy to deform, or soft.

Example 4

A solid copper sphere of 0.5m^3 volume is placed 30.5 m below the ocean surface where the pressure is $3.00 \times 10^5 \text{ N/m}^2$. What is the change in volume of the sphere? The bulk modulus for copper is $14 \times 10^{10} \text{ N/m}^2$.

Solution

The change in volume is given by,

$$\begin{aligned}\Delta V &= -\frac{V_0 P}{B} \\ &= -\frac{(0.500\text{m}^3)(3.00 \times 10^5\text{N/m}^2)}{14 \times 10^{10}\text{N/m}^2} \\ &= -1.1 \times 10^{-6} \text{ m}^3\end{aligned}$$

Example 4

The average depth of Indian Ocean is about 3000 m. Calculate the fractional compression, $\Delta V/V$, of water at the bottom of the ocean, given that the bulk modulus of water is $2.2 \times 10^9 \text{ N m}^{-2}$. (Take $g = 10 \text{ m s}^{-2}$)

Solution

The pressure exerted by a 3000 m column of water on the bottom layer

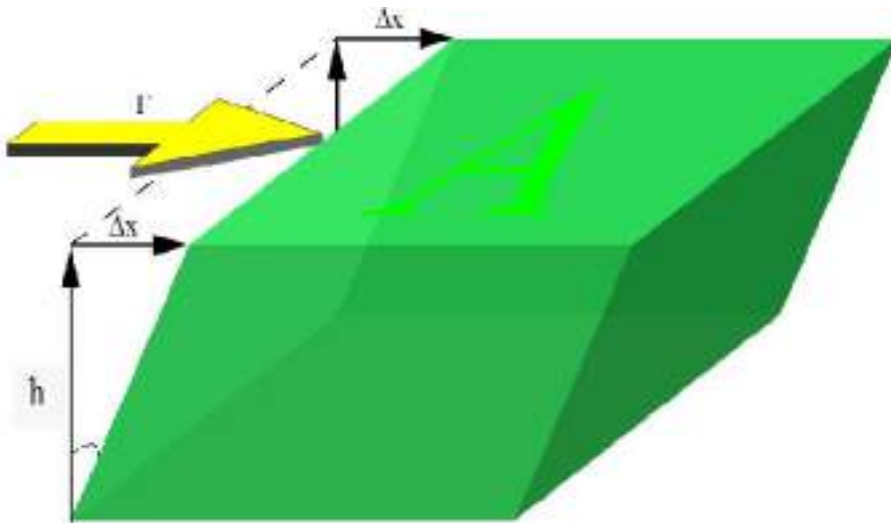
$$\begin{aligned}p &= h \rho g = 3000 \text{ m} \cdot 1000 \text{ kg m}^{-3} \cdot 10 \text{ m s}^{-2} \\ &= 3 \cdot 10^7 \text{ kg m}^{-1} \text{ s}^{-2} \\ &= 3 \cdot 10^7 \text{ N m}^{-2}\end{aligned}$$

Fractional compression $\Delta V/V$, is

$$\frac{\Delta V}{V} = \frac{\text{Stress}}{B} = \frac{(3 \times 10^7 \text{ N.m}^{-2})}{2.2 \times 10^9 \text{ N.m}^{-2}} = 1.36 \times 10^{-2}$$

3. Shear Modulus

Consider a **cube fixed to the surface** as in the figure. A tangential force F is applied at the top of the cube, a distance h above the bottom.



The magnitude of this force F times the height h of the cube would normally cause a torque to act on the cube to rotate it. However, since the cube is not free to rotate, the body instead becomes deformed and changes its shape. The tangential force thus causes a

change in the shape of the body that is measured by the angle ϕ , called the **angle of shear**. We can also relate ϕ to the linear change from the original position of the body by noting from the figure that

$$\tan \phi = \frac{\Delta x}{h}$$

Because the deformations are usually quite small, as a first approximation the $\tan \phi$ can be replaced by the angle ϕ itself, expressed in radians. Thus,

$$\phi = \frac{\Delta x}{h}$$

this equation represents the **shearing strain** of the body.

$$\text{Shearing strain} = \frac{\Delta x}{h} = \phi$$

$$\text{Shearing stress} = \frac{\text{Force}}{\text{Area}} = \frac{F}{A}$$

The proportionality constant that relates these two quantities together is the ratio of shear stress to shear strain — **Shear modulus or modulus of rigidity (R)**.

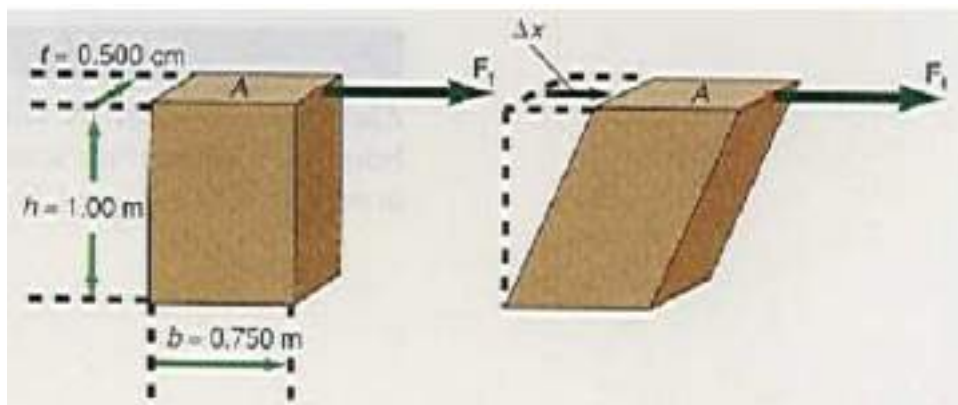
$$\text{Shear modulus (R)} = \frac{\text{Shearing stress}}{\text{Shearing strain}} = \frac{F/A}{\Delta x/h}$$

Shear modulus dimensions = the dimensions of stress = $ML^{-1}T^{-2}$

Shear modulus units: (MKS system) $\rightarrow kg \cdot m^{-1} \cdot s^{-2} \equiv N \cdot m^{-2}$
 (cgs system) $\rightarrow gm \cdot cm^{-1} \cdot s^{-2} \equiv dyne \cdot cm^{-2}$

Example 5

A sheet of copper 1.00 m high, base length 0.750 m, and 0.500 cm thick is acted on by a tangential force of 50,000 N, as shown in the below figure. The value of R for copper is $4.20 \times 10^{10} \text{ N/m}^2$. Find (a) the shearing stress, (b) the shearing strain, and (c) the linear displacement Δx .



Solution

a. The area that the tangential force is acting over is

$$\begin{aligned} A &= bt = (0.750 \text{ m})(5.00 \times 10^{-3} \text{ m}) \\ &= 3.75 \times 10^{-3} \text{ m}^2 \end{aligned}$$

where b is the length of the base and t is the thickness of the copper sheet shown in the figure. The shearing stress is

$$\frac{F}{A} = \frac{50.000 \text{ N}}{3.75 \times 10^{-3} \text{ m}^2} = 1.33 \times 10^7 \text{ N/m}^2$$

b. The shearing strain is

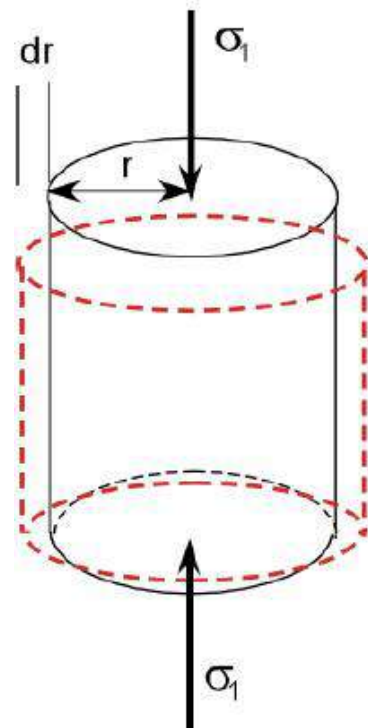
$$\begin{aligned} \phi &= \frac{F/A}{R} = \frac{1.33 \times 10^7 \text{ N/m}^2}{4.20 \times 10^{10} \text{ N/m}^2} \\ &= 3.17 \times 10^{-4} \text{ rad} \end{aligned}$$

c. The linear displacement Δx is

$$\begin{aligned} \Delta x &= h\phi = (1.00 \text{ m})(3.17 \times 10^{-4} \text{ rad}) \\ &= 3.17 \times 10^{-4} \text{ m} = 0.317 \text{ mm} \end{aligned}$$

Poisson's ratio

One more important physical property is Poisson's relationship. When an object is compressed (or extended) it is observed that it expands (or contracts) perpendicular to the direction of the applied stress. The ratio of the transverse strain to the axial strain is Poisson's ratio (σ).



Consider a rectangular block in the following figure with original length (l) and diameter (d) is subjected to a tensile force (axial force) then, the length is increased by (Δl) and the diameter is decreased by (Δd).

There is exist two strains

The **transverse** strain = $-\Delta d/d = -\Delta r/r$

The **axial** strain = $\Delta l/l$

Poisson's ratio is given by:

$$\sigma = \frac{\text{transverse strain}}{\text{axial strain}}$$

$$\sigma = \frac{(-\Delta d/d)}{(\Delta l/l)}$$

$$\sigma = \frac{-\Delta r/r}{\Delta l/l}$$

If the force compresses Poisson's ratio it is given by:

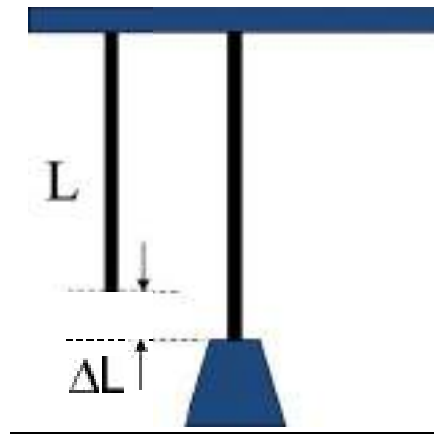
$$\sigma = \frac{\Delta r/r}{-\Delta l/l}$$

Poisson's ratio dimensions = dimensionless

Poisson's ratio units: has no unit.

Example 6

A steel wire 10 m long and 2 mm in diameter is attached to the ceiling and a 200-N weight is attached to the end. What is the applied stress?



Solution

$$L=2\text{m}$$

$$\text{Diameter} = d = 2\text{mm} = 2 \times 10^{-3}\text{m}$$

$$r = d/2 = 1 \times 10^{-3}\text{m}$$

$$A = \pi r^2 = 3.14(0.001)^2$$

$$F = 200\text{ N}$$

$$\text{Stress} = ?$$

$$\text{Stress} = F/A$$

$$\text{stress} = 200 / 3.14(0.001)^2$$

$$= 63.69 \times 10^{-6} \text{N.m}^{-2}$$

Example 7

A 10 m steel wire stretches 3.08 mm due to the 200 N loads. What is the longitudinal strain

Solution

Given

$$l = 10\text{m}$$

$$\Delta l = 3.08\text{mm}$$

$$F = 200\text{N}$$

Longitudinal strain

$$= \Delta l / l = (3.08\text{mm}) \times 10^{-3} / 10\text{m} = 0.308 \times 10^{-3}$$

Problems

1. A coil spring stretches 4.00 cm when a mass of 500 g is suspended from it. What is the force constant of the spring?
2. A coil spring stretches by 2.50 cm when a mass of 750 g is suspended from it. (a) Find the force constant of the spring. (b) How much will the spring stretch if 800 gm is suspended from it?
3. An aluminum wire has a diameter of 0.850 mm and is subjected to a force of 1000 N. Find the stress act on the wire.
4. A copper wire experiences a stress of $5.00 \times 10^3 \text{ N/m}^2$. If the diameter of the wire is 0.750 mm, find the force acting on the wire.
5. A steel wire, 1.00 m long, has a diameter of 1.50 mm. If a mass of 3.00 kg is hung from the wire, by how much will it stretch?
6. A copper wire, 1.00 m long, has a diameter of 0.750 mm. When an unknown weight is suspended from the

wire it stretches 0.200 mm. What was the load placed on the wire?

7. A copper block, 7.50 cm on a side, is subjected to a tangential force of 3.5×10^3 N. Find the angle of shear.

8. A cube of lead 15.0 cm on a side is subjected to a uniform pressure of 5.00×10^5 N/m². By how much does the volume of the cube change?

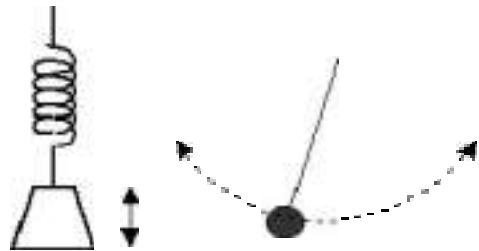
Chapter (3)

Oscillatory motion

It is a very common type of motion used for time keeping.

Examples:

The pendulum (a kid on a swing) and weights on springs



Properties of oscillatory motion: Amplitude, frequency, and period

Pull down a weight on a spring and it oscillates around the equilibrium position.

Push on a pendulum (a kid in a swing) and it oscillates around the equilibrium position. The pendulum has the following characteristics that we can measure:

1) Amplitude

It is defined as the maximum displacement from the equilibrium position. This is from the center of the swing to the end of the swing.

There is an alternate term: **peak-to-peak amplitude**.

This is defined as the distance from one end of the swing to the other. This is convenient when you are not sure where the center is. The regular amplitude is half the peak-to-peak amplitude.

The **units** of amplitude are expressed in meters for pendulums and ocean waves.

2) Frequency

It is the number of oscillations per second.

Its unit called hertz (Hz). The units of Hertz are 1/seconds sometimes written s^{-1} or /s.

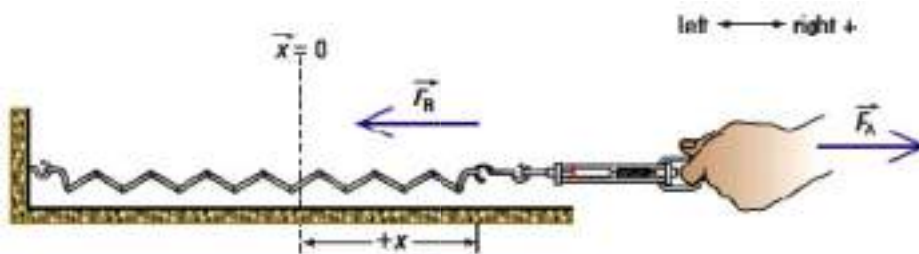
3) Periodic time

This is the time interval necessary for a complete motion.

Simple Harmonic Motion (SHM):

Simple harmonic motion is a special case of oscillatory motion that occurs when the restoring force is proportional to the displacement.

A simple harmonic oscillator, a force pull a spring to a positive displacement ($\vec{x}$) as shown in the Figure. While you hold it there, the spring exerts an equal and opposite force in your hand, as described by Newton's third law. However, this force is to the left, in the negative direction, and attempts to restore the spring to its equilibrium position. This force is called the **restoring force**.



The restoring force always acts in a direction opposite to the displacement. Therefore, Hooke's law is properly written with a negative sign when representing the *restoring force*.

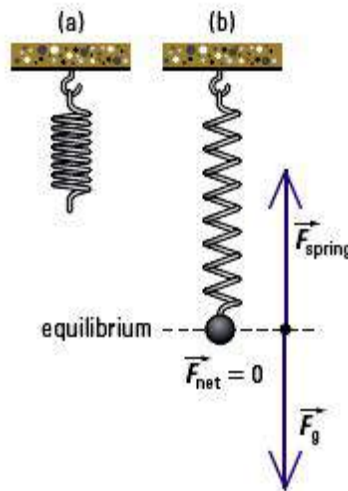
$$\vec{F} = -k\vec{x}$$

In this case, while the spring is held in this position, the applied force and the restoring force have equal magnitudes but opposite directions, so the net force on

the system is zero. In the next section, a mass will be attached to the spring and it will slide on a frictionless horizontal surface. The restoring force will be the only force in the system and will give rise to a repetitive back-and-forth motion called ***simple harmonic motion***.

S. H. M. of Vertical Mass-spring Systems

In the figure below, the spring in (a) has no mass attached. In (b), the spring stretches until the force exerted by the mass is equal and opposite to the force of gravity, and equilibrium is reached.



The net force (or restoring force) is the vector sum of the force of gravity and the tension of the spring. In this case, it is zero.

Once a simple harmonic oscillator is set in motion, it will slowly come to rest because of friction *unless* a force is continually applied. We will examine these conditions in on resonance. SHM is repetitive and predictable, so we can state the following:

- *The restoring force acts in the opposite direction to the displacement.*
- *At the extremes of SHM, the displacement is at its maximum and is referred to as the amplitude. At this point, force and acceleration are also at their maximum, and the velocity of the object is zero.*
- *At the equilibrium position, the force and acceleration are zero, and the velocity of the object is at its maximum.*

$$F = -kx$$

$$ma_x = -kx$$

$$m \frac{d^2x}{dt^2} = -kx$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

$$\frac{d^2x}{dt^2} = -\omega^2x$$

The acceleration is proportional to the displacement and **the minus sign indicates that the acceleration and the displacement are oppositely directed**. This relation is the defining characteristic of simple harmonic motion and can be used to identify systems that will exhibit it:

In simple harmonic motion, both the acceleration and thus the net force, are proportional to the displacement from the equilibrium position, but oppositely directed from it.

This is the condition of simple harmonic motion

S. H. M. AND CIRCULAR MOTION

A relation exists between simple harmonic motion and circular motion with constant speed v . Imagine a particle moving with constant speed in a circle of radius as shown in the figure. Its angular displacement relative to $+x$ the direction is given by

$$\theta = \omega t + \delta$$

Where: δ is the angular displacement at time $t = 0$ and is the angular speed $\omega = v/A$ of the particle.

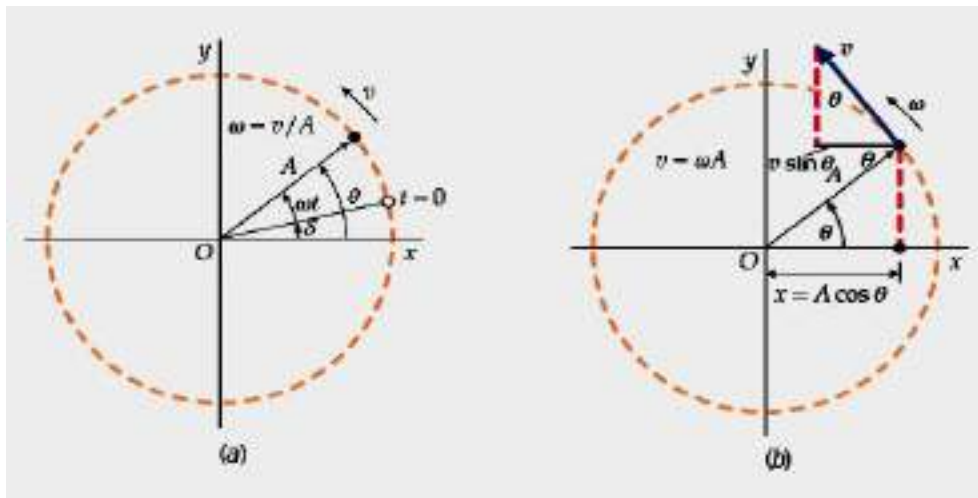
The component of the particle's position (Figure -b) is

$$x = A \cos \theta = A \cos(\omega t + \delta)$$

This is the same as Equation (2) for simple harmonic motion.

The speed of a particle moving in a circle is $r\omega$, where: r is the radius. For the particle in Figure -b, $r = A$ so its speed is $A\omega$. The projection of the velocity vector onto the axis gives $v_x = -v \sin \theta$ Substituting for and gives

$$v_x = -v \sin \theta = -\omega A \sin(\omega t + \delta)$$



Also we can get the acceleration of the moving particle by the same manner

$$a_x = \frac{dv_s}{dt} = \frac{d(-\omega A \sin(\omega t + \delta))}{dt}$$

$$a_x = -\omega^2 x$$

This equation represents the acceleration of particle moving in circular path with angular velocity ω

$$\omega = \sqrt{\frac{a_x}{x}}$$

And the frequency

$$f = \frac{1}{2\pi} \sqrt{\frac{a_x}{x}}$$

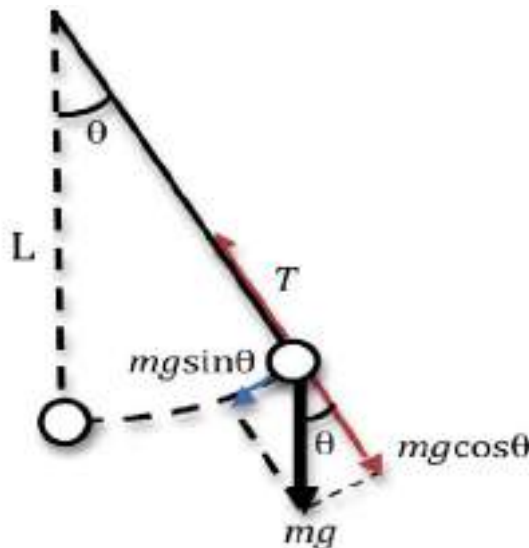
The time period of a complete cycle is

$$T = 2\pi \sqrt{\frac{x}{a_x}}$$

The Simple Pendulum

A simple pendulum consists of a light string of length is L fixed from its upper end, and the lower end attached with a bob of mass m as shown in the figure.

For a small displacement the pendulum wire makes a small angle ϕ , so $\sin \phi = \phi$. If the small displacement is x from the equilibrium position. Since $\sin \phi = x/L = \phi$



The forces acting on the bob are:

- The force T exerted by the string (tension of the string).
- The gravitational force (weight of the ball) $= mg$.

The tangential component of the gravitational force, $mg\sin\theta$, always acts toward $\theta = 0$, opposite the displacement. Therefore, the tangential force is a restoring force, and we can apply Newton's second law for motion in the tangential direction:

$$F = -mg\sin\theta$$

$$ma = -mg\sin\theta$$

$$m \frac{d^2s}{dt^2} = -mg\sin\theta$$

$$\frac{d^2s}{dt^2} = -g\sin\theta$$

where: s is the bob's displacement measured along the arc and the minus sign indicates that the tangential force acts toward the equilibrium (vertical) position.

Because $\tan\theta = \frac{s}{L}$, for the small displacement x from the equilibrium position. So $\tan\theta = x/L = \theta$

So,

$$L \frac{d^2\theta}{dt^2} = -g \sin\theta$$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L} \sin\theta$$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L} \theta$$

But;

$$\frac{d^2x}{dt^2} = -\frac{k}{m} x$$

$$\omega^2 = \frac{g}{L}$$

the angular frequency ω is

$$\omega = \sqrt{\frac{g}{L}}$$

and the periodic time is given by:

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{L}{g}}$$

In other words, the period and frequency of a simple pendulum *depend only on **the length of the string and the acceleration due to gravity.***

Momentum

What Is Momentum??? If you're standing at the bottom of a hill and you're faced with the option of stopping a runaway semi-truck or stopping a runaway bicycle, you'd probably choose to stop the bike, right? The reasoning behind this is that the semi-truck has more **momentum** than the bike. Momentum simply means **mass in motion**.

The semi-truck has a large momentum because it is very massive, but it also has a large speed, which influences momentum, as well. The bike also has momentum because it has a large speed, but because its mass is less than that of the truck; its momentum is also less.

This relationship can be described in an equation:

momentum = mass x velocity

$$P = m v$$

where p is momentum, m is mass (in kg) and v is velocity (in m/s). You may remember that **velocity** is speed with direction, so if an object has a large speed, it also has a large velocity. This also means that an object at rest does not have momentum. The velocity of an object at rest is zero, so there's no movement. In order for an object to have momentum, it must be moving!

An important thing to note about momentum is that it is a **vector quantity**. This means that it has both magnitude and direction. Velocity is also a vector quantity because it has both of these components, and fortunately for us, the direction of the momentum is the same as the direction of the velocity vector.

Angular momentum,

It is a measure of the amount of rotation an object has, taking into account its mass, shape and speed. It is

a vector quantity that represents the product of a body's rotational inertia and rotational velocity about a particular axis. For a rigid body the angular momentum can be expressed as the product of the body's moment of inertia, I , (i.e., a measure of an object's resistance to changes in its rotation velocity) and its angular velocity, ω .

$$\mathbf{L} = I \times \omega$$

where: $\mathbf{L}$ is the angular momentum vector whose direction will be in the direction of the angular velocity vector ω for simple cases. The direction of $\mathbf{L}$ can be found using a right-hand algorithm. The line of the vector $\mathbf{L}$ is parallel to the axis of rotation, and the direction is such that one sees clockwise rotation as looking along the axis.

For the case of an object that is small compared with the radial distance to its axis of rotation, such as a planet orbiting in an ellipse around the Sun, the angular momentum can be approximated as the cross product of

its linear momentum, mv (v being equal to ωr) and its position relative to the point about which it is rotating, r . Thus, the angular momentum, L , of a particle with respect to some point of origin is as follows:

$$L = r \times mv$$

Angular momentum is conserved if there is no unbalanced torque acting on the system. Then the product of the moment of inertia and the angular velocity will remain a conserved quantity of the system,

$$I_f \omega_f = I_i \omega_i$$

where the subscripts i and f refer to the initial and final states of the system and I and ω represent the moment of inertia and the angular velocity of the system.

Example 1

Let us consider the case of the skater who goes into a spin. Assume that at the beginning of the spin her moment of inertia is $10.0 \text{ kg} \cdot \text{m}^2$ and the angular velocity is 5.00 rad. /sec. If the moment of inertia of the skater reduced to $2.00 \text{ kg} \cdot \text{m}^2$, what is her final angular velocity?

Solution

$$I_f \omega_f = I_i \omega_i$$

$$(10.0 \text{ kg} \cdot \text{m}^2)(5.00 \text{ rad./sec}) = (2.00 \text{ kg} \cdot \text{m}^2) \omega_f$$

$$\omega_f = 25.0 \text{ rad./sec}$$

Planetary Motion

The apparent zigzag motion of the planets across the night sky puzzled human observers of the heavens for many centuries. We now have the mental constructs to discuss planetary motion in a simple way. Consider the sun as a large, massive body that produces a gravitational field. The strength of the field is given by the law of gravitational attraction,

$$F = G \frac{Mm}{r^2}$$

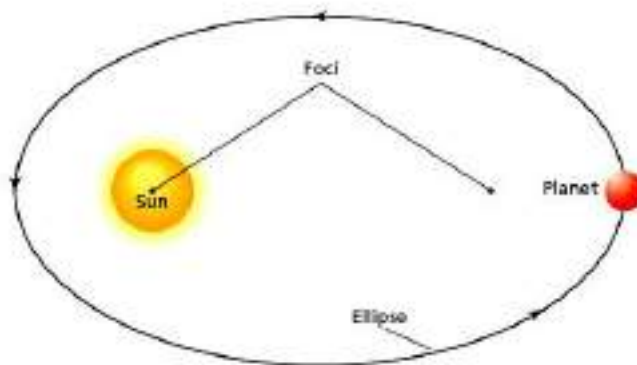
Where: F is the magnitude of the force of gravitational attraction, G is the universal gravitational constant, M is the mass of the sun, m is the mass of the object in the sun's gravitational field, and r is the distance between the object and the sun. The gravitational force acts

toward the sun. It applies no torque to an object moving in a path around the sun. So the angular momentum of objects moving in closed orbits around the sun is constant,

$$L = I\omega = (mr^2)(v/r) = mvr = \text{constant}$$

Kepler's first law:

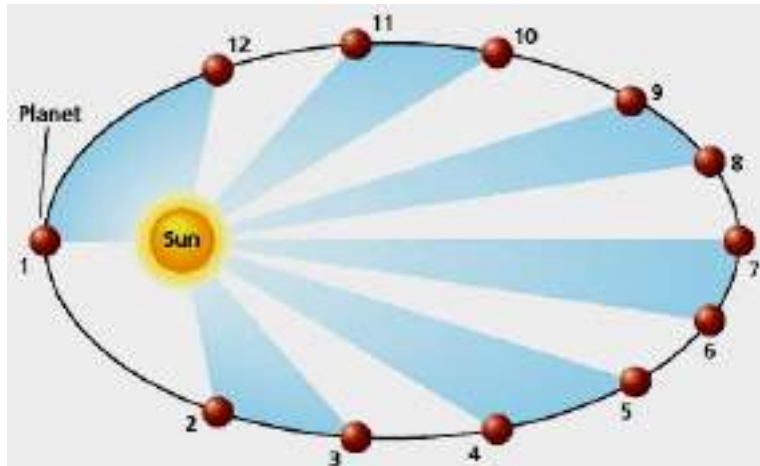
All planets move in elliptical orbits with the Sun at one focus of the ellipse.



Planets orbit the Sun in elliptical orbits with the Sun at one focus.

Kepler's second law:

The line joining the sun and a planet sweeps out equal areas in equal times, i.e. planets don't move at constant speed.



A planet moves fastest when it is close to the Sun and slowest when it is farther from the Sun. Equal areas are swept out in equal amounts of time.

Let us assume that the planets travel in circular orbits. The result we will derive is also true for elliptical orbits, but the mathematics is simpler for circular orbits.

For circular motion v and r are perpendicular to each other and can be drawn as the two sides of a right triangle. The area of such a triangle is proportional to the product of velocity times radius, but vr is proportional to the angular momentum, so

$$\text{Area} \propto vr = \frac{L}{m} = \text{constant}$$

Kepler's third law:

The ratio of the cube of the radius of the orbit to the square of the time for the planet to make a complete cycle around the sun is a constant for all the planets.

$$\frac{r^3}{T^2} = \text{constant}$$

The period of time required for a planet to complete one cycle around the sun is the circumference of its orbit divided by its speed of travel.

$$T = \frac{2\pi r}{v}$$

where: r is the distance of the planet from the sun and v is the speed of the planet. We can obtain an expression for the speed of the planet in its orbit by equating the centripetal force to the gravitational force.

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

It follows that the square of the velocity is inversely proportional to the radius of the orbit.

$$v^2 = \frac{GM}{r}$$

This expression for v^2 can be substituted into the square of Equation (10) to obtain an equation for the period T in terms of the radius r :

$$T^2 = \frac{4\pi^2 r^2}{v^2} = \frac{4\pi^2 r^2}{(GM/r)} = \frac{4\pi^2 r^3}{GM}$$

This equation can be rearranged to show that the ratio r^3/T^2 is a constant, equal in size to the product of the universal gravitational constant times the mass of the sun divided by $4\pi^2$:

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

Thus, if the periods of the planets are T_A and T_B , and their average distances from the Sun are r_A and r_B , Kepler's third law can be expressed as follows.

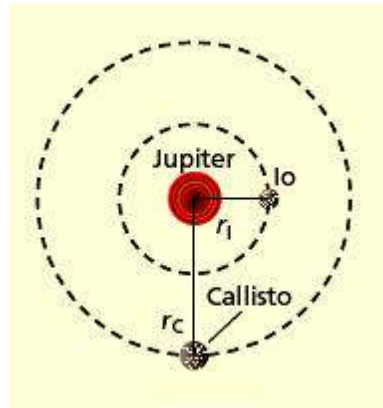
$$\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{r_A}{r_B}\right)^3$$

The squared quantity of the period of object A divided by the period of object B, is equal to the cubed quantity of object A's average distance from the Sun, divided by object B's average distance from the Sun.

Note that the first two laws apply to each planet, moon, and satellite individually. The third law, however, relates the motion of several objects about a single body. For example, it can be used to compare the planet's distances from the Sun to their periods about the Sun. It also can be used to compare distances and periods of the Moon and artificial satellites orbiting Earth.

Example

Galileo measured the orbital sizes of Jupiter's moons using the diameter of Jupiter as a unit of measure. He found that Io, the closest moon to Jupiter, had a period of 1.8 days and was 4.2 units from the center of Jupiter. Callisto, the fourth moon from Jupiter, had a period of 16.7 days. Using the same units that Galileo used; predict Callisto's distance from Jupiter.



Solution

$$T_C = 16.7 \text{ days}$$

$$r_C = ?$$

$$T_1 = 1.8 \text{ days}$$

$$r_1 = 4.2 \text{ units}$$

Solve Kepler's third law for r_C .

$$\left(\frac{T_C}{T_1}\right)^2 = \left(\frac{r_C}{r_1}\right)^3$$

$$r_C^3 = r_1^3 \left(\frac{T_C}{T_1}\right)^2$$

$$r_C = \sqrt[3]{r_1^3 \left(\frac{T_C}{T_1}\right)^2}$$

$$r_C = \sqrt[3]{(4.2 \text{ AU})^3 \left(\frac{16.7 \text{ days}}{1.8 \text{ days}}\right)^2}$$

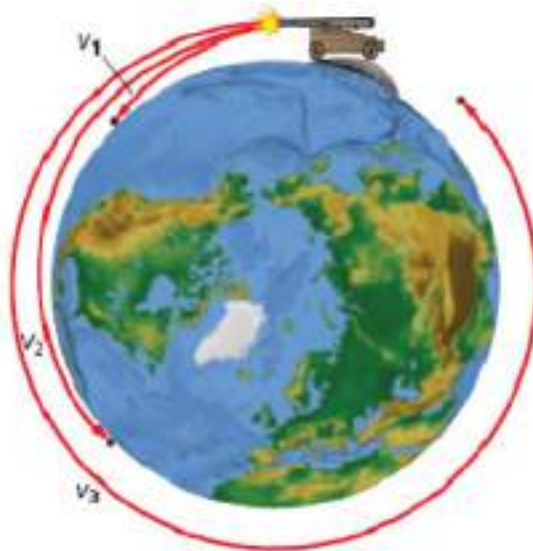
$$r_C = \sqrt[3]{6.4 \times 10^3 \text{ AU}^3}$$

$$r_C = 19 \text{ AU}$$

Substitute $r_1 = 4.2\text{AU}$, $T_C = 16.7 \text{ days}$, $T_1 = 1.8 \text{ days}$

Orbits of Planets and Satellites

Newton used a drawing similar to the one shown in the **figure** to illustrate a thought experiment on the motion of satellites. Imagine cannon, perched high atop a mountain, firing a cannonball horizontally with a given horizontal speed. The cannonball is a projectile, and its motion has both vertical and horizontal components. Like all projectiles on Earth, it would follow a parabolic trajectory and fall back to the ground.



If the cannonball's horizontal speed were increased, it would travel farther across the surface of Earth, and still fall back to the ground. If an extremely powerful cannon were used, however, the cannonball would travel all the way around Earth, and keep going. It would fall toward Earth at the same rate that Earth's surface curves away. In other words, the curvature of the projectile would continue to just match the curvature of Earth, so that the cannonball would never get any closer or farther away from Earth's curved surface. The cannonball would, therefore, be in orbit.

Newton's thought experiment ignored air resistance.

For the cannonball to be free of air resistance, the mountain on which the cannon are perched would have to be more than 150 km above Earth's surface. By way of comparison, the mountain would have to be much taller than the peak of Mount Everest, the world's tallest mountain, which is only 8.85 km in height. A cannonball launched from a mountain that is 150 km above Earth's surface would encounter little or no air resistance at an

altitude of 150 km, because the mountain was above most of the atmosphere. Thus, a cannonball or any object or satellite at or above this altitude could orbit Earth for a long time

A satellite in an orbit that is always the same height above Earth moves in uniform circular motion. Recall that its centripetal acceleration is given by $a_c = v^2/r$. Newton's second law, $F_{net} = ma_c$, can thus be rewritten as $F_{net} = mv^2/r$. If Earth's mass is m_E , then this expression combined with Newton's law of universal gravitation produces the following equation:

$$G \frac{m_E m}{r^2} = \frac{mv^2}{r}$$

Solving for the speed of a satellite in circular orbit about Earth, v , yields the following.

Speed of a Satellite Orbiting Earth

$$v = \sqrt{\frac{Gm_E}{r}}$$

The speed of a satellite orbiting Earth *is equal to the square root of the universal gravitational constant times the mass of Earth, divided by the radius of the orbit.*

A satellite's orbital period

A satellite's orbit around Earth is similar to a planet's orbit about the Sun. Recall that the period of a planet orbiting the Sun is expressed by the following equation:

$$T = 2\pi \sqrt{\frac{r^3}{Gm_s}}$$

Thus, the period for a satellite orbiting Earth is given by the following equation.

Period of a Satellite Orbiting Earth

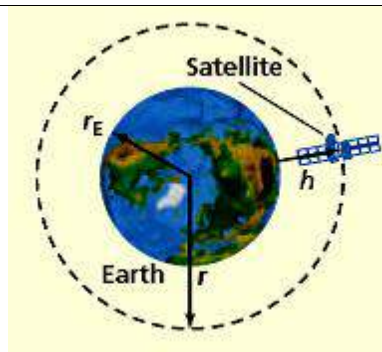
$$T = 2\pi \sqrt{\frac{r^3}{Gm_E}}$$

The period for a satellite orbiting Earth *is equal to 2π times the square root of the radius of the orbit cubed, divided by the product of the universal gravitational constant and the mass of Earth.*

The equations for the speed and period of a satellite can be used for any object in orbit about another. The mass of the central body will replace m_E orbiting body and the central body. If the mass of the central body is much greater than the mass of the orbiting body, then r is equal to the distance between the centers of the orbiting body and the central body. Orbital speed, v , and period, T , are independent of the mass of the satellite. Are there any factors that limit the mass of a satellite?

Example

Assume that a satellite orbits Earth 225 km above its surface. Given that the mass of Earth is 5.97×10^{24} kg and the radius of Earth is 6.38×10^6 m, what are the satellite's orbital speed and period?



Solution

$$h = 2.25 \times 10^5 \text{ m}$$

$$v = ?$$

$$r_E = 6.38 \times 10^6 \text{ m}$$

$$T = ?$$

$$m_E = 5.97 \times 10^{24} \text{ kg}$$

$$G = 6.67 \times 10^{-11} \text{ N.m}^2/\text{kg}^2$$

Determine the orbital radius by adding the height of the satellite's orbit to Earth's radius.

$$r = h + r_E$$

Substitute

$$h = 2.25 \times 10^5 \text{ m}, \quad r_E = 6.38 \times 10^6 \text{ m}$$

$$= 2.25 \times 10^5 \text{ m} + 6.38 \times 10^6 \text{ m} = 6.61 \times 10^6 \text{ m}$$

Solve for the speed.

$$v = \sqrt{\frac{Gm_E}{r}}$$

$$v = \sqrt{\frac{(6.67 \times 10^{-11} \text{ N.m}^2/\text{kg}^2)(5.97 \times 10^{24} \text{ kg})}{6.61 \times 10^6 \text{ m}}}$$

$$v = 7.76 \text{ m/s}$$

Solve for the period.

$$T = 2\pi \sqrt{\frac{r^3}{Gm_E}}$$

$$T = 2\pi \sqrt{\frac{(6.61 \times 10^6 \text{ m})^3}{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2)(5.97 \times 10^{24} \text{ kg})}}$$

$$T = 5.35 \times 10^3 \text{ s}$$

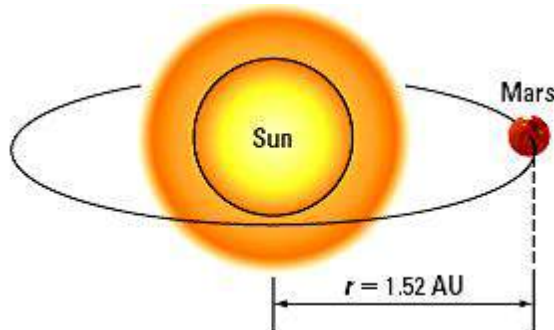
This is approximately 89 min, or 1.5 h.

Astronomical Units

A more suitable measurement for astronomical distances has been adopted. It is called the astronomical unit (AU). One astronomical unit is the mean orbital distance from Earth to the Sun (the length of the semi-major axis of Earth's orbit). This is a more manageable unit to use. For example, Neptune is only 30.1 AU away from the Sun on average. Kepler's constant, using units of years (a) and AU, can be determined as follows:

$$k = \frac{T_E^2}{r_E^3} = \frac{(1\text{a})^2}{(1\text{AU})^3} = 1\text{a}^2/\text{AU}^3$$

Ex. Mars has an orbital radius of 1.52 AU. What is its orbital period?



Mars has a mean orbital radius of 1.52 AU (not drawn to scale). Kepler's third law can be used to determine its period.

Start with the equation for determining Kepler's constant:

$$k = \frac{T_{Mars}^2}{r_{Mars}^3}$$

Where: T_{Mars} is the orbital period of Mars, and r_{Mars} is the mean orbital radius of Mars.

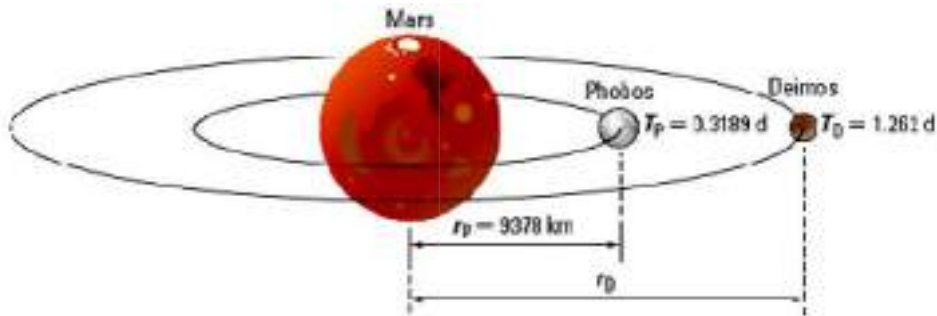
$$T_{Mars}^2 = kr_{Mars}^3$$

$$T_{Mars} = \sqrt{\left(1 \frac{a^2}{AU^3}\right) (1.52AU)^3} = 1.87a$$

The orbital period of Mars is 1.87 Earth years. In other words, it takes 1.87 Earth years for Mars to go around the Sun once.

Example

Mars has two moons, Deimos and Phobos (the figure below). Phobos has an orbital radius of 9378 km and an orbital period of 0.3189 Earth days. Deimos has an orbital period of 1.262 Earth days. What is the orbital radius of Deimos?



The orbital period and radius of Phobos can be used with the orbital period of Deimos to determine its orbital radius (not drawn to scale).

Solution

$$T_P = 0.3189 \text{ d} \quad r_P = 9378 \text{ km} \quad T_D = 1.262 \text{ d}$$

$$\frac{T_D^2}{r_D^3} = \frac{T_P^2}{r_P^3} \rightarrow r_D^3 = \frac{T_D^2}{T_P^2} r_P^3$$

$$r_D^3 = \frac{(1.262 \text{ d})^2}{(0.3189 \text{ d})^2} (9376 \text{ km})^3$$

$$r_D^3 = 1.2916 \times 10^{13} \text{ km}^3$$

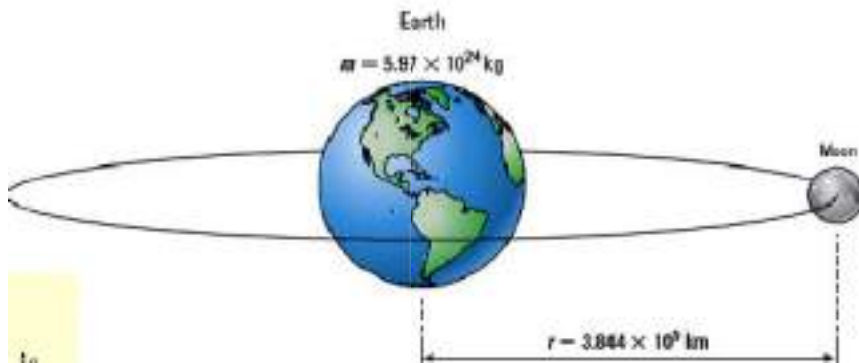
$$r_D = \sqrt[3]{1.2916 \times 10^{13} \text{ km}^3}$$

$$r_D = 2.346 \times 10^4 \text{ km}$$

The orbital radius of Deimos is $2.346 \times 10^4 \text{ km}$. This answer is reasonable since a moon with a larger orbital period than Phobos will also have a larger orbital radius.

Example

Earth's Moon is $3.844 \times 10^5 \text{ km}$ from Earth (Figure 3.12). Determine the orbital speed of the Moon.



Solution

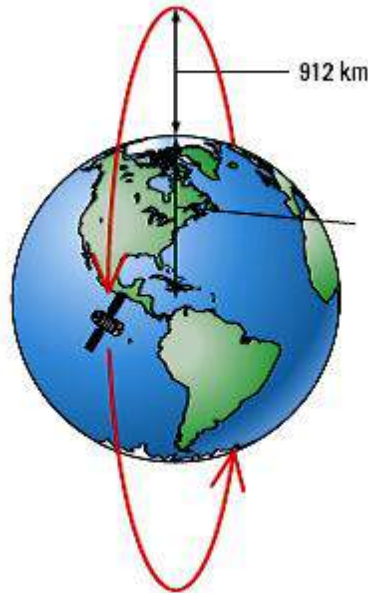
$$v = \sqrt{\frac{Gm_E}{r}}$$

$$v = \sqrt{\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2)(5.97 \times 10^{24} \text{ kg})}{3.844 \times 10^8 \text{ m}}}$$

$$v = 1.02 \times 10^3 \text{ m/s}$$

Example

Land Sat is an Earth-imaging satellite that takes pictures of Earth's ozone layer and geological features. It orbits Earth at the height of 912 km (Figure 3.13). What are its orbital speed and its period?



The radius of Land Sat's orbit must be measured from the centre of Earth. To do this, add Earth's radius to the satellite's height above the planet's surface. Then determine the speed and period of the satellite.

Solution

$$r = r_{\text{Earth}} + 912\,000\text{ m} = (6.38 \times 10^6\text{ m}) + (9.12 \times 10^5\text{ m}) \\ = 7.292 \times 10^6\text{ m}$$

$$v = \sqrt{\frac{Gm_{\text{Earth}}}{r}}$$

$$v = \sqrt{\frac{(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97 \times 10^{24}\text{ kg})}{7.292 \times 10^6\text{ m}}}$$

$$v = 7.390 \times 10^3 \text{ m/s}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi(7.292 \times 10^6 \text{ m})}{7.390 \text{ m/s}} = 6.20 \times 10^3 \text{ s}$$

The speed of the satellite is $7.39 \times 10^3 \text{ m/s}$ ($2.66 \times 10^4 \text{ km/h}$). It orbits Earth once every $6.20 \times 10^3 \text{ s}$ (103 minutes).

Problems

1. Use Kepler's third law to determine the orbital period of Jupiter. Its orbital radius is 5.203 AU.
[11.87 Earth years]
2. Pluto takes 90 553 Earth days to orbit the Sun. Use this value to determine its mean orbital radius.
[39.465 AU]
3. A piece of rocky debris in space has a mean orbital distance of 45.0 AU. What is its orbital period?[302 Earth years]
4. Using 365 days as Earth's period and its solar orbit radius as 1.00 astronomical unite (AU), find the distance of an asteroid from the sun that has a period of 2920 days1 [Astronomical Unit = 149 598 000 kilometers] . [4.0 AU]

Chapter (4)

Hydrostatics

4-1 Three phases of matter

From our everyday experience, we have become familiar with the fact that matter occurs in three different *forms-solid, liquid, and gas*. Under ordinary conditions stone, iron, copper, and chalk, for example, are solids; water, oil, and mercury are liquids; air, hydrogen, and carbon dioxide are gases. Each one of these forms is called a *phase*. At times it is difficult to distinguish clearly between the solid and the liquid phases, as in a material such as tar which flows under the action of a force at ordinary temperatures. Even where the different phases are clearly recognizable, materials undergo a phase change under different conditions of temperature and pressure. We shall confine our discussion to the application of the principles of mechanics to bodies which remain in the same phase.

Liquids and *gases* are sometimes grouped together as *fluids* because they *flow* very readily under the application of an external force, while solids do not. A solid has a definite size and a definite shape, and these change only slightly under the application of external forces. For this reason it is possible to study the statics of *solids* by characterizing them as *rigid* bodies. *Liquids*, on the other hand, possess a definite size or volume but change their shape very readily. Liquids at rest generally take the shape of the containing vessel. If the containing vessel has a volume greater than that of the liquid put into it, there will be a *free surface* at the top of the liquid. A gas differs from a liquid in that a gas has neither size nor shape. A quantity of gas placed in a container will completely fill that container, there is no free surface. The volume of the gas is the volume of the container

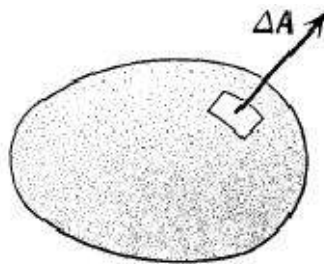
4-2 Pressure

There is a difference in the manner in which a force is applied to a fluid and the way it is applied to a solid. A

force can be supported by a single point of a free solid, but it can only be supported by a surface of an enclosed fluid. In a discussion of the results of the application of forces to fluids, it is convenient to introduce a new term called **pressure**. If a force F is applied to the surface of a fluid and acts over an area A perpendicular to it, then the average pressure P is defined as

$$P=F/A \quad (4-1)$$

The pressure may be expressed in dynes per square centimeter, in pounds per square foot, in Newton's per square meter, or in any other appropriate set of units. Pressure is a scalar quantity.



An area vector A is directed along the outward normal to the surface.

When a fluid is under pressure, it exerts a force on any surface which contains the fluid.

Equation (8-1), which describes the average pressure, is not quite complete, for, while relating the pressure and the area, it must be accompanied of the force. We may make the equation more complete by considering *area as a vector quantity*. An element of area ΔA may be described as a vector whose magnitude is the numerical value of the area and whose direction is perpendicular to the surface of the area element, as shown in Figure 8-1. If the element of area is part of a closed surface, it is conventional to choose the *outward normal* to the surface as the direction of the area vector. With this convention we may write

$$\Delta F = P \Delta A$$

For the force ΔF exerted by a fluid of pressure P against any surface element of area ΔA .

The pressure may vary from point to point within a fluid. We speak of the pressure at a point within a volume of fluid, meaning that we imagine the point to be

surrounded by a small container and divide the total force exerted by the fluid against the walls of the container by the area of the container. Following the usual limiting processes of the calculus, we examine this quotient as the volume of the container gets smaller and smaller, and we call its limiting value the pressure at the point

4-3 Density

In discussing distributions of matter such as solids or fluids, it is convenient to define a quantity called the density ρ (rho) as the mass per unit volume or,

$$\rho = M/V$$

Thus the mass of a homogeneous body of material of volume V is given by $m = \rho V$. The units in which density is expressed are the ones appropriate to the system of units being used. In the c.g.s system of units the density is expressed in grams per cubic centimeter; in the m.k.s system of units the density is expressed in kilograms per

cubic meter; in the British gravitational system of units the density is expressed in slugs per cubic foot.

It is common engineering practice to use the word "density" to express the weight per unit volume in pounds per cubic foot. We distinguish this quantity from the density defined above by referring to the weight per unit volume as the *weight density*.

4-4 Pressure Due to Weight of a Liquid

Let us consider the equilibrium conditions of a small element of liquid volume submerged within the body of the liquid, as shown in Figure 4-2. Since the volume element is at rest, the horizontal forces exerted upon this volume element by the surrounding liquid must have a zero resultant. In the vertical direction we note that there must be a difference in the forces exerted on the top and bottom faces of the volume element sufficient to support the weight of the liquid within that volume element.

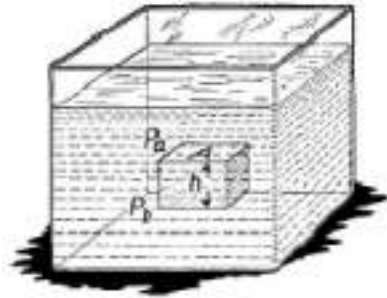


Fig. 4-2

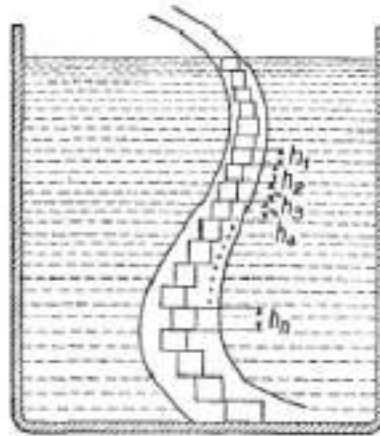


Fig. 4-3

If the pressure at the level of the top face is P_a and the pressure at the level of the bottom face is P_b , the downward force exerted on the top face of the volume element by the surrounding liquid is $P_a A$, while the upward force exerted forces must against the bottom face is $P_b A$. The difference between these is equal to the weight of the liquid contained within the element of height h .

Thus we have

$$P_b A - P_a A = A h \rho g$$

and, dividing by A ,

$$P_b - P_a = h \rho g$$

Thus the pressure difference between two adjacent level surfaces in the liquid is given by $h\rho g$, where h represents the vertical distance between the two surfaces.

In the event that we wish to find the pressure within a liquid confined in a tube of irregular shape, such as the S-shaped tube of Figure 4-3, we may imagine the liquid to be subdivided into a succession of volume elements, one atop the other, each of which contributes a small increment of pressure, depending on its vertical height, to the pressure at the depth h .

Thus the pressure at all points at a given depth under the surface of a liquid depends upon the depth but not upon the shape of the container;

All points at the same horizontal level surface within a body of liquid are at the same pressure, as long as the liquid is at rest.

In fact, this statement is often taken as the definition of a level surface. If the pressure on the surface of a liquid

is taken as zero, then the pressure P at any point a distance h below this level is given by:

$$P = \rho gh$$

Example 1

Determine the pressure at the bottom of a column of mercury 70 cm high

Solution

From Equation 4-5) we have

$$P = h\rho g.$$

We substitute the value of $h = 70$ cm, the density ρ of mercury = 13.6 gm/cm^3 , and $g = 980 \text{ cm/sec}^2$, and obtain

$$P = 70 \text{ cm} \times 13.6 \text{ gm/cm}^3 \times 980 \text{ cm/sec}^2.$$

$$P = 9.33 \times 10^5 \text{ dynes.cm}^2$$

4-5 Pressure in a Confined Liquid

In addition to the pressure due to its weight, a confined liquid may be subjected to an additional pressure by the application of an external force. Suppose the liquid is in a cylinder, as shown in Figure 4-4, and that a tight fitting piston is placed on the surface of the liquid. If a force F is applied to the piston, it will remain in practically the same position, since the compressibility of liquids is very small. If A is the area of the piston, this external force produces a pressure $P = F / A$ at the surface of the liquid

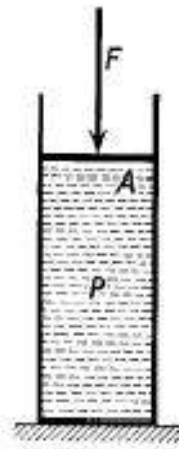


Fig. 4-4 A pressure P is produce in a confined liquid by the application of force F on a piston of area A in contact with the liquid.

As we have seen from the previous discussion, as we examine the pressure at increasing depth under the surface of the liquid, each element of volume adds a contribution to the pressure which is due to its own weight to the pressure at the top surface of that volume, so that the pressure P due to the external force is transmitted throughout every part of the liquid and acts on all surfaces in contact with the liquid. This is sometimes known as ***Pascal's principle*** and may be stated as follows:

Whenever the pressure in a confined liquid is increased or diminished at any point, this change in pressure is transmitted equally throughout the entire liquid.

The operation of the hydraulic press, the hydraulic brakes of a car, and the hydraulic lift is based upon Pascal's principle. The hydraulic press, sketched in Figure 4-5, consists essentially of two connected cylinders one of small cross-sectional area A_1 , the other of large cross-sectional area A_2 , each fitted with a piston. A liquid,

usually oil or water, is supplied to it from a reservoir. By exerting a force F on the small piston, an additional pressure $P = F / A_1$ produced. This pressure is transmitted throughout the liquid and hence acts on the larger piston of area A_2 . The force that can be exerted by the larger piston is then PA_2 . If the hydraulic press is designed to lift a weight W , then

$$W = P A_2 = F (A_2 / A_1)$$

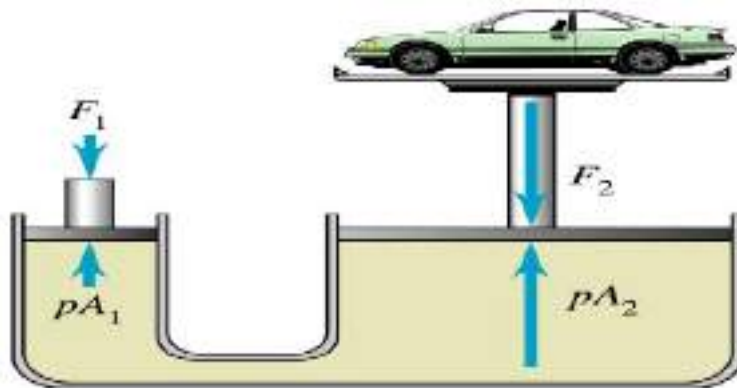


Fig. 4-5 Hydraulic pressure; the pressure P is the same everywhere throughout the liquid, if the pressure due to the weight of the liquid is negligible.

The hydraulic press may be considered as a simple machine in which the force exerted by the machine

divided by the force exerted on the machine, is equal to the ratio of the areas of its pistons; thus

$$W/F = A_2/A_1$$

A force-distributing system such as that used to operate the brakes of a car offers the very great advantage that the force applied on each of the brakes is automatically equal. Furthermore, the force is transmitted with very little mechanical movement of the hydraulic link.

Example

The small piston of a hydraulic lift has an area of 0.20 m^2 . A car weighing $1.20 \times 10^4 \text{ N}$ sits on a rack mounted on the large piston. The large piston has an area of 0.90 m^2 . How large a force must be applied to the small piston to support the car?

Solution

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$F_1 = \left(\frac{A_1}{A_2} \right) F_2 = \left(\frac{0.2 \text{ m}^2}{0.90 \text{ m}^2} \right) (1.20 \times 10^4 \text{ N})$$

$$F_1 = 2.7 \times 10^3 \text{ N}$$

4-6 Atmospheric Pressure

The atmosphere is a layer of air surrounding the earth; its thickness has been estimated at about 500 600 mi. The density of the air decreases with increasing altitude. Since air has weight, this layer of air produces a pressure, called the earth. The atmospheric pressure varies from day to day by about 5 per cent, the variations often accompanying changes in the weather the pressure of the air is measured by a *barometer*, which often consists of an evacuated tube inserted in a dish of mercury, as shown in Figure 4-6.

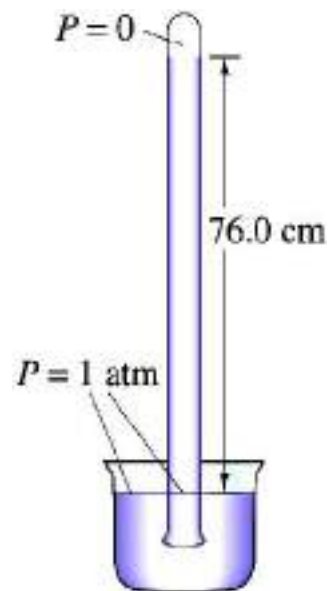


Fig. 4-6

The atmosphere exerts a pressure P on the open surface of the mercury in the dish, and this is transmitted to the liquid in the tube **Fig.4-6** Mercury barometer. This pressure is balanced by the pressure due to the mercury in the tube at a height h above the open surface of the dish. To look at the barometer another way, we recall that the pressure is always the same at any level surface in a liquid. Consider the level surface defined by the surface of the mercury in the dish. Outside the barometer tube the pressure at this surface is entirely due to the atmosphere, so that the pressure of the mercury here is atmospheric pressure. Hence the pressure of the mercury within the tube at the level of the surface of the mercury in the dish is also atmospheric pressure. Knowing the density of mercury and the height to which the column of mercury rises within the evacuated barometer tube, we can calculate the atmospheric pressure

In meteorology other units of pressure are used; they are mentioned here only for the sake of completeness.

*A pressure of **one bar** is defined as one million dynes per square centimeter. A mill bar is one thousandth of a bar.*

Standard atmospheric pressure can thus be expressed as 1,013 mill bars.

4-7 Pressure Gauges: Manometers

We have already encountered one type of pressure gauge, the mercury barometer. A second type of pressure gauge is the *aneroid* barometer, which consists of a partially evacuated cylindrical box made of corrugated metal, as shown in Figure 4-7.

The difference in pressure between the inside of the chamber and the atmosphere causes the faces of the box to deflect, and this is balanced by a steel spring. The motion of the upper surface of the box is coupled to a pointer whose indication is read on a scale. Such a gauge

is often used as a pressure altimeter on board aircraft, for atmospheric pressure varies with altitude, and so can be used as a measure of altitude. Another type of gauge, a *Bourdon* gauge, consists essentially of a flattened brass tube, closed at one end and bent into circular form. When a fluid under pressure is admitted to the open end of the tube, the tube straightens slightly, and this motion is coupled to a pointer whose deflection is calibrated.

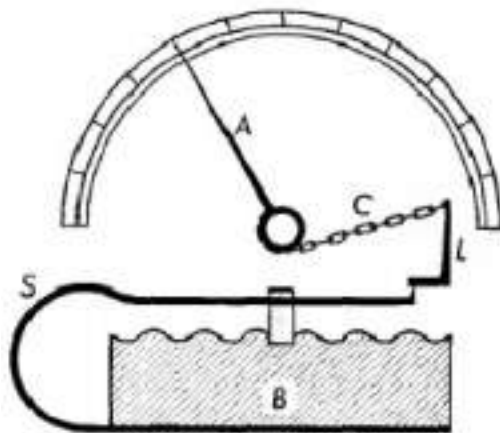


Fig. 4-7. Aneroid barometer; B is a box containing air at reduced pressure, S is a spring, L a lever, C a chain, and A a pointer

Modern Manometer

The simplest type of pressure gauge is the *open-tube manometer* illustrated in Figure 4-8, which consists essentially of a bent tube of transparent material with

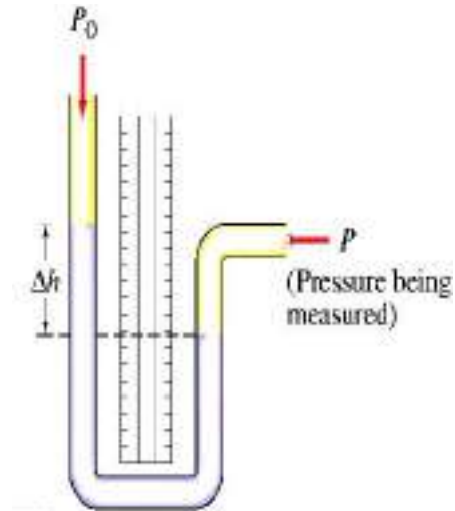


Fig. 4-8

both arms vertical. One end A is open to the atmosphere, and the other end B is connected to the vessel in which the pressure is to be measured. The tube is partially filled with a liquid of density ρ . Suppose that the pressure within the vessel is greater than the atmospheric pressure, causing the liquid to rise in the column exposed to the atmosphere

Again we note that the pressure within a body of liquid is the same everywhere along a level surface. Choosing as the level surface of reference that of the lower liquid surface of the manometer, we observe that the pressure within the vessel P must be equal to the pressure of the atmosphere plus the pressure due to a column of liquid

of height h , the difference in level between the liquid surfaces in the two arms of the manometer tube. Thus we have

$$P = P_{atm.} + \rho g h$$

In many technical applications it is the difference in pressure between the inside of a container and the atmosphere which is of importance, rather than the pressure itself. This is the case in a steam boiler, or in a gas line, and in many other applications of the manometer. The difference $P - P_{atm.}$ is called the ***gauge pressure***. The pressure P is called the *absolute pressure*.

A barometer thus reads the absolute pressure of the atmosphere, while the height h of an open-tube manometer is a measure of the gauge pressure. The gauge pressure of the atmosphere itself is, of course, zero.

4-8 Archimedes' Principle

The fact that some objects float in water while others sink to the bottom has been known for centuries; Archimedes (287-212 B.C.) was the first to discover the principle underlying these phenomena. To understand Archimedes' principle it is necessary to consider the forces acting on a body totally immersed in a liquid, as shown in Figure 8-9. There is a downward force on the body equal to its own weight W , and, in addition, there is a buoyant force B on the body, which acts in the upward direction. This can be understood by imagining the volume now occupied by the body to be occupied instead by an equal volume of liquid. This volume of liquid would have been in equilibrium, which means that its weight would have been supported by the action of the rest of the liquid. This support comes from the difference in pressure between the top and the bottom of this volume.

Hence, no matter what material occupies this volume, there will be a force upward on it equal to the weight of

Fig. 4-9 Buoyant force on a the liquid displaced.

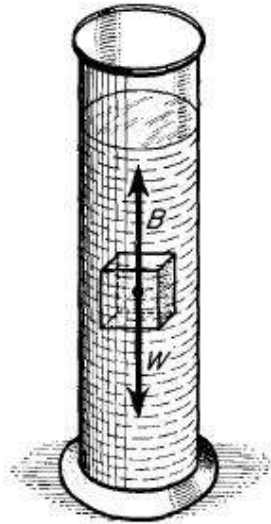


Fig. (4-9) *Buoyant force on a block immersed in a liquid.*

Archimedes' principle is a generalization of the result obtained above; it states that:

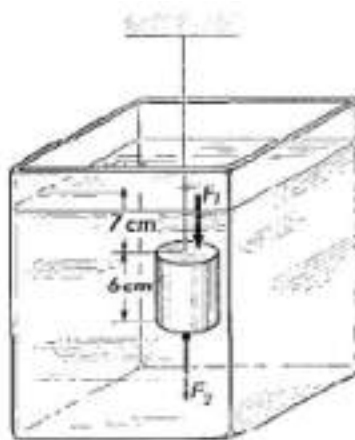
Any object partly or completely immersed in a fluid is buoyed up by a force equal to the weight of the fluid displaced.

The principle is applicable to both liquids and gases. A body completely immersed in a fluid will sink if its weight is greater than the buoyant force and will rise if the buoyant force is greater than its weight.

Equilibrium will be established when the weight of the liquid displaced is equal to the weight of the body. Thus a solid body which is less dense than water floats on the surface of the water, part below the water and part above the water. A ship afloat, for example, displaces its own weight of water. The weight of a ship is frequently expressed in terms of the weight of the water it displaces, and one speaks of ships with 10,000 tons displacement, and so on. There is usually a definite water line painted on a ship, indicating the limit to which a ship may be submerged and still be safe.

Example 1

A cylinder of brass 6 cm high and 4 cm² m cross-sectional area is suspended in water by means of a string so that its upper surface is 7 cm below the surface of the water, as shown in Figure 8-10. Determine (a) the force acting on the bottom of the cylinder, (b) the force acting on the top of the cylinder, and (c) the buoyant force acting on this cylinder

Solution

(a) The force F_1 acting on the top of the cylinder is that due to the pressure of the water above it and is

$$F_1 = P_1 A = h_1 \rho g A$$

$$F_1 = 7 \text{ cm} \times 1 \text{ gm/cm}^3 \times 980 \text{ cm/sec.}^2 \times 4 \text{ cm}^2$$

$$F_1 = 27,440 \text{ dynes}$$

This force pushes down on the cylinder.

(b) The force F_2 acting on the bottom of the cylinder is due to the pressure of the water above it. Since the depth of the water at the bottom of the cylinder is 13 cm, this force is

$$F_2 = P_2 A = h_2 \rho g A$$

$$F_2 = 13 \text{ cm} \times 1 \text{ gm/cm}^3 \times 980 \text{ cm/sec.}^2 \times 4 \text{ cm}^2$$

$$F_2 = 50,960 \text{ dynes}$$

Acting upward on the cylinder

(c) The buoyant force B is the net force upward caused by the difference in pressures in the liquid. The forces which act on the walls of the cylinder are all directed horizontally, and their resultant is zero, as can be seen from the symmetry of the figure. Therefore, the buoyant force is simply the difference between the two vertical forces F_1 and F_2 , thus

$$B = F_2 - F_1 = 50,960 - 27,440$$

$$= 23,520 \text{ dynes}$$

And acts upward

It is interesting to compare this buoyant force with the weight of water displaced. The volume of the cylinder is $h \times A = 24 \text{ cm}^3$. This is also the volume.

Of water displaced. The weight of this displaced water is

$$W = mg = \rho V g$$

$$W = 1 \text{ gm/cm}^3 \times 24 \text{ cm}^3 \times 980 \text{ cm/sec.}^2$$

$$W = 23,520 \text{ dynes,}$$

This is in agreement with the earlier calculation

Example 2

A block of aluminum is attached to a balance. When suspended in air, the balance reads 250 gm. When the aluminum block is lowered so that it is completely immersed in water, the balance reads 160 gm. When the aluminum block is lowered so that it is completely immersed in alcohol, the balance reads 180 gm. The density of water is 1 gm/cm³. Determine (a) the density of aluminum, and (b) the density of alcohol.

Solution

(a) The buoyant force of the water is the difference between the weight of the aluminum block in air and its weight when immersed in water; that is,

$$\begin{aligned} B &= 250 \times 980 \text{ dynes} - 160 \times 980 \text{ dynes} \\ &= 90 \times 980 \text{ dynes.} \end{aligned}$$

Thus the aluminum block displaces a mass of 90 gm of water. The volume of water displaced is

$$V = m/\rho = 250(\text{gm})/90(\text{gm/cm}^3) = 90\text{cm}^3$$

And this is equal to the volume of aluminum. Thus the density of aluminum is

$$\rho = m/v = 250 \text{ gm} / 90 \text{ cm}^3$$

$$\rho = 2.78 \text{ gm/cm}^3$$

(b) The amount of alcohol displaced by the aluminum block may be found

$$\begin{aligned} B &= 250 \times 980 \text{ dynes} - 180 \times 980 \text{ dynes} \\ &= 70 \times 980 \text{ dynes,} \end{aligned}$$

so that the mass of alcohol displaced by the aluminum block is 70 gm.

The volume of alcohol displaced is the volume of the aluminum block, which we have found to be equal to 90 cm³. Thus the density of alcohol is

$$\rho = 70 \text{ gm} / 90 \text{ cm}^3$$

$$\rho = 0.78 \text{ gm/cm}^3$$

4-9 Specific Gravity

It is sometimes convenient to refer the mass of a given object to the mass of an equal volume of water. The term *specific gravity* is used to denote the magnitude of this quotient, and since the specific gravity is a property of the material of which the object is made rather than of the size or shape of the object, we may conveniently define the specific gravity as the *ratio of the density of a body to the density of water*. Thus water has a specific gravity of 1. The specific gravity is a pure number and is independent of the system of units used to measure the density. Since the density of water in the cgs system of units is 1 gm/cm^3 the specific gravity of a substance has the same numerical value as its density in the cgs system.

From the definition of specific gravity, we see that a body of specific gravity less than 1 will float in water, while a body of specific gravity greater than 1 will sink

The specific gravity of liquids is commonly determined by use of an instrument called the *hydrometer*, which is usually made in the form of a cylinder with a weighted bulb at one end, as shown in Figure 4-10. The depth to which the hydrometer will sink in a liquid depends on its specific gravity, so the hydrometer may be provided with a calibrated scale to read specific gravity directly, or to read some property

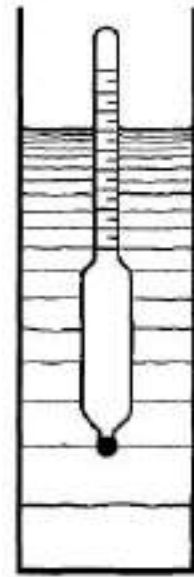


Fig. 4-10

associated with specific gravity which is of immediate interest. Since the freezing point of mixture of antifreeze and water is determined by the fraction of antifreeze in the solution, the hydrometer may be directly calibrated to read the freezing temperature of the mixture. *A hydrometer is a simple instrument used to measure the specific gravity of a liquid by indicating how deeply the instrument sinks in the liquid.*

Chapter (5)

Surface Tension

One of most important phenomena in nature which describes:

- Water climbing up trees.
- Shape of rain drops.
- Some insects moving above water.



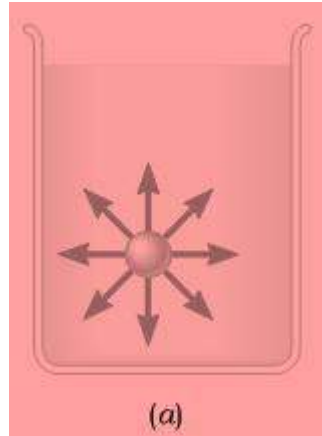
These observations suggest that the surface of a liquid acts like an elastic membrane covering the liquid or in a state of tension. As we shall soon show, surface tension is due to intermolecular attraction.

Molecular basis for surface tension

The fact that a liquid surface is in a state of tension can be explained by the intermolecular forces. The figure illustrates the molecular basis for surface tension by considering the attractive forces that molecules in a liquid exert on one another.

➤ **In the bulk of the liquid (part A),**

- A particular molecule is surrounded by an equal number of molecules on all sides.
- The average distance apart of the molecules is such that the attractive forces balance the repulsive forces.
- The average intermolecular force between A and the surrounding molecules is zero.



➤ **In the surface of the liquid (part B),**

- Since there are no molecules of the liquid above the surface, the molecule on the surface experiences a net attractive force pointing toward the liquid interior. This net attractive force causes the liquid surface to contract toward the interior.



Thus, if the particle on B is displaced very slightly upward, a resultant attractive force F on it, now has to be overcome. It follows that if all molecules in the surface were removed to infinity, a definite amount of work is needed to bring molecules from the bulk to the surface. Consequently molecules in the surface have more potential energy than those in the bulk.

Surface area (shape of drop)

The potential energy of any system in stable equilibrium is a minimum. Thus under surface tension forces, the area of a liquid surface will have the least number of molecules in it, that is, the surface area of a given volume of liquid is a minimum. So, if the liquid is not acted upon by external forces, a liquid sample forms a sphere, which has the minimum surface area for a given volume. Nearly spherical drops of water are a familiar view.

Surface tension definition (units and dimensions)

The surface tension is denoted by the Greek letter gamma (γ), sometimes called the coefficient of **surface tension is defined as:**



The magnitude F of the force exerted parallel to the surface of a liquid divided by the length L of the line over which the force acts:

$$\gamma = \frac{F}{L}$$

- SI Unit of Surface Tension: N/m

- Surface Tension dimensions = $\frac{\text{dimension of force}}{\text{dimension of length}}$

$$= \frac{MLT^{-2}}{L} = MT^{-2}$$

The surface tension coefficient of depends on the temperature of the liquid and on the medium on the other side of the surface.

Capillary Action

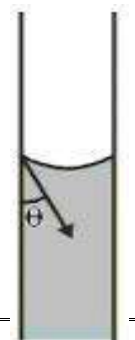
We have seen that surface tension arises because of the intermolecular forces of attraction that molecules in a liquid exert on one another. These forces, which are **between like molecules**, are called **cohesive forces**. A liquid, however, is often in contact with a solid surface, such as glass. Then additional forces of attraction come into play. They occur **between molecules of the liquid and molecules of the solid surface** and, being between unlike molecules, are called **adhesive forces**.

Consider a tube with a very small diameter, which is called a capillary. When a capillary, open at both ends, is inserted into a liquid, the result of the competition between cohesive and adhesive forces can be observed.

The figure shows ***a glass capillary inserted into water.***

In this case,

- The adhesive forces are stronger than the cohesive forces, so that the water



molecules are attracted to the glass more strongly than to each other.

- The result is that the water surface curves upward against the glass.
- It is said that the water “wets” the glass.

The surface tension leads to a force F acting on the circular boundary between the water and the glass. This force is oriented at an angle ϕ , which is determined by the competition between the cohesive and adhesive forces. The vertical component of F pulls the water up into the tube to a height h . At this height the vertical component of F balances the weight of the column of water of length h .

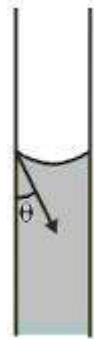
The angle ϕ called “**contact angle**”, and is always measured through the liquid. The angle of contact between two given surfaces varies largely with their



freshness and cleanliness. The angle of contact between water and very clean glass is zero, but when the glass is not clear the angle of contact is $< 90^\circ$.

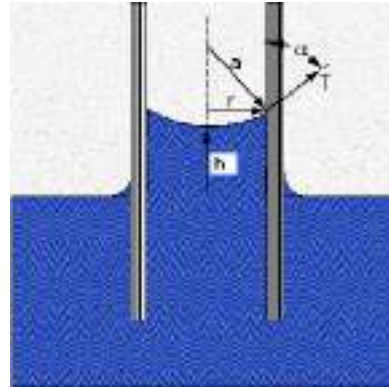
The figure, shows ***glass capillary inserted into mercury,***

- The adhesive forces are weaker than the cohesive forces.
- The mercury atoms are attracted to each other more strongly than they are to the glass. As a result, the mercury surface curves downward against the glass and the mercury do not “wet” the glass.
- The surface tension leads to a force F , the vertical component of which pulls the mercury down a distance h in the tube.
- The angle of contact is $> 90^\circ$.



Determination of surface tension coefficient by capillary tube:

when capillary tube is immersed in liquid; there will be a raised or depressed column of liquid inside it. The case of a raised column is shown on the figure.



The upward component of the surface tension force (upward force) will balance the weight of the liquid column (downward force).

We know that: $\gamma = \frac{F}{L}$; and the length of liquid in contact with the glass is $2\pi r$ So:

$$F = 2\pi r \gamma$$

The **upward component** of the surface tension force is given by:

$$F = 2\pi r \gamma \cos \theta$$

The downward force (weight of the liquid column) is given by:

$$mg = \rho Vg$$

The volume of the liquid = $\pi r^2 h$

Thus, the weight of the liquid = $\rho \pi r^2 h g$

At balance

$$2\pi r \gamma \cos\theta = \rho \pi r^2 h g$$

Hence, the surface tension of liquid can be expressed as:

$$\gamma = \frac{r \rho g h}{2 \cos\theta}$$

Example 1

A capillary tube with an inside diameter of 250 μm can support a 100 mm column of liquid that has a density of 930 kg.m^{-3} . The observed contact angle is 15° . Find the surface tension of the liquid.

Solution

$$\begin{aligned}
 T &= \frac{\rho g r h}{2 \cos \theta} \\
 &= \frac{930 \times 9.8 \times (125 \times 10^{-6})}{2 \cos 15} \\
 &= 0.06 \text{ Nm}^{-1}
 \end{aligned}$$

Pressure inside a soap bubble and a liquid drop

Anyone who's blown up a balloon has probably noticed that the air pressure inside the balloon is greater than on the outside. For instance, if the balloon is suddenly released, the greater inner pressure forces the air out, propelling the balloon much like a rocket. The reason for the greater pressure is that the tension in the stretched rubber tends to contract the balloon. To counteract this tendency, the balloon has a greater interior air pressure acting to expand the balloon.

A soap bubble (see the figure below -a) has two spherical surfaces (inside and outside) with a thin layer of liquid in-between. Like a balloon, the pressure inside a soap bubble is greater than that on the outside. As we will see

shortly, this difference in pressure depends on the surface tension γ of the liquid and the radius R of the bubble. For the sake of simplicity, let's assume that there is no pressure on the outside of the bubble ($P_o = 0$). Now, imagine that the stationary soap bubble is cut into two halves. Being at rest, each half has no acceleration and so is in equilibrium. According to Newton's second law of motion, a zero acceleration implies that the net force acting on each half must be zero ($\Sigma F = 0$). We will now use this equilibrium relation to obtain an expression relating the interior pressure to the surface tension and the radius of the bubble.

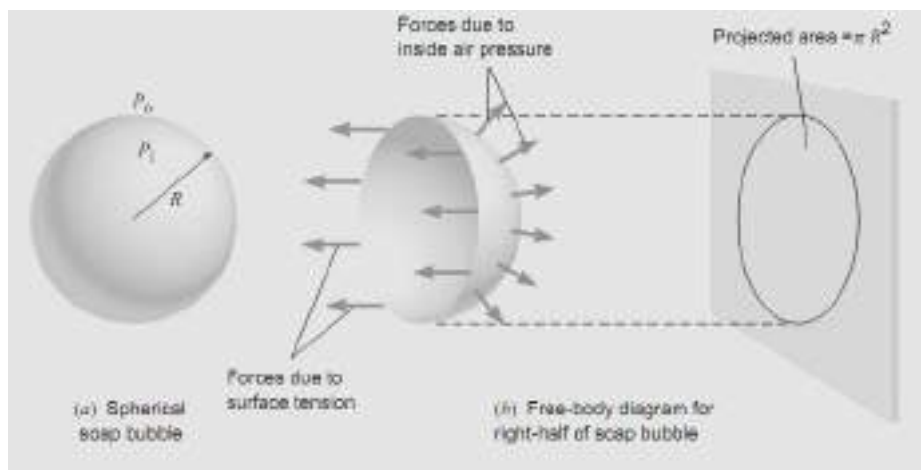


Figure -b shows a free-body diagram for the right half of the bubble, on which two forces act.

1. **First**, there is the force due to the **surface tension in the film**. This force is exerted on the right half of the bubble by the left half. The surface tension force points to the left and acts all along the circular edge of the hemispherical film. The magnitude of the force due to each surface of the film is the product of the surface tension γ and the circumference ($2\pi R$) of the circular edge, or $\gamma (2\pi R)$. The total force due to the inner and outer surfaces is twice this amount or $-2 \gamma (2\pi R)$. We have included the minus sign to denote that this force points to the left in the drawing. We have also assumed the film to be sufficiently thin enough that its inner and outer radii are nearly the same.
2. **Second**, there is a force caused by the **air pressure inside the bubble**. At each point on the surface of the bubble, the force due to the air pressure is

perpendicular to the surface and is directed outward. Figure 7b shows this force at six points on the surface. When these forces are added to obtain the total force due to the air pressure, all the components cancel, except those pointing to the right. The total force due to all the components pointing to the right is equal to the product of the pressure P_i inside the bubble times the circular cross-sectional area of the hemisphere, or $P_i (\pi R^2)$.

Using these expressions for the forces due to the surface tension and air pressure, we can write Newton's second law of motion as:

$$\Sigma F = 0$$

$$\underbrace{-2\gamma(2\pi R)}_{\text{Force due to surface tension}} + \underbrace{P_i(\pi R^2)}_{\text{Force due to pressure inside bubble}} = 0$$

Solving this equation for the pressure inside the bubble gives $P_i = 4\gamma / R$. In general, the pressure P_o outside the bubble is not zero. However, this result still gives the difference between the inside and outside pressures, so that we have

$$P_i - P_o = \frac{4\gamma}{R} \quad (\text{spherical soap bubble})$$

This result tells us that the difference in pressure **depends on the surface tension and the radius of the sphere**. What is surprising is that a greater pressure exists inside a smaller soap bubble (smaller value of R) than inside a larger one.

A spherical drop of liquid, like a drop of water, has only one surface, rather than two surfaces, for there is no air within it. Thus, the force due to the surface tension is only one-half as large as that in a bubble. Consequently, the difference in pressure between the inside and outside of a liquid drop is one-half of that for a soap bubble:

$$P_i - P_o = \frac{2\gamma}{R} \quad (\text{spherical liquid drop})$$

This result also holds for a spherical bubble in a liquid, such as a gas bubble inside a glass of beer. However, the surface tension γ is that of the surrounding liquid in which the trapped bubble resides.

Example 2

(a) A student, using a circular loop of wire and a pan of soapy water, produces a soap bubble whose radius is 1.0 mm. The surface tension of the soapy water is $\gamma = 2.5 \times 10^{-2}$ N/m. Determine the pressure difference between the inside and outside of the bubble. (b) The same soapy water is used to produce a spherical droplet whose radius is one-half that of the bubble, or 0.50 mm. Find the pressure difference between the inside and outside of the droplet.

Solution

(a) The pressure difference, $P_i - P_o$, between the inside and outside of the soap bubble is given by

$$P_i - P_o = \frac{4\gamma}{R} = \frac{4(2.5 \times 10^{-2} \text{ N/m})}{1.0 \times 10^{-3} \text{ m}} = \boxed{1.0 \times 10^2 \text{ N/m}^2}$$

(b) The pressure difference, $P_i - P_o$, between the inside and outside of the drop is given by:

$$P_i - P_o = \frac{2\gamma}{R} = \frac{2(2.5 \times 10^{-2} \text{ N/m})}{0.50 \times 10^{-3} \text{ m}} = \boxed{1.0 \times 10^2 \text{ N/m}^2}$$

Problems

1. Two soap bubbles ($\gamma = 0.025 \text{ N/m}$) have radii of 2.0 and 4.5 mm. For each bubble, determine the difference between the inside and outside pressures.
2. A small bubble of air in water ($\gamma = 0.073 \text{ N/m}$) has a radius of 0.10 mm. Find the difference in pressures between the inside and outside of the bubble.

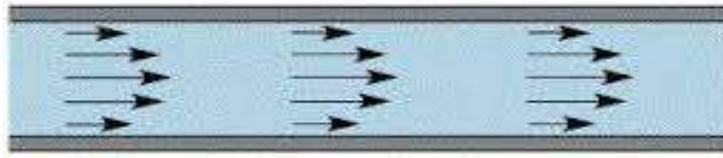
Chapter (6)

Viscosity

when real fluids flow they have a certain amount of internal friction called **viscosity**. So, **Viscosity is defined as**; the resistance of a fluid to flow. It exists in both liquids and gases and is essentially a friction force between different layers of fluid as they move past one another. In liquids the viscosity is due to the cohesive forces between the molecules whilst in gases the viscosity is due to collisions between the molecules.

Laminar flow:

It sometimes known as streamline flow; occurs when a fluid flows in parallel layers, with no disruption between the layers. Put some transparent glycerol in a tube and then put in some colored glycerol. Let them flow. From the colored glycerol's shape change, we can see the flow velocity of glycerol is different in different places.



- The speed is smaller near the wall of the tube and flow velocity is the biggest in the center of the tube. This means that viscous fluid flows in different layers. We call it Laminar Flow.
- When fluid flows in laminar flow, there is internal friction or viscous force between two close layers. The internal friction is caused by the interaction forces between molecules. The internal friction is bigger in liquids than in gases.

The basic formula for the frictional force 'F' in a liquid was first suggested by **NEWTON**. He found that the frictional force is:

- ✓ directly proportional to the area of the surface of liquid (A).
- ✓ directly proportional to the velocity gradient. If v_1 , v_2 are the velocities of two layers separated by

distance h . The velocity gradient between the two layers can be expressed as; $\left(\frac{v_1 - v_2}{h}\right)$ in unit $(\text{m/s})/\text{m}$ or s^{-1} .

So,

$$F \propto A \times \text{velocity gradient}$$

The constant of proportionality for the fluid is called the coefficient of viscosity η :

$$F = \eta A \times \text{velocity gradient}$$

The greater the coefficient of viscosity η , the greater the force required to move the plate at a velocity v .

Definition, Units and Dimensions of coefficient of viscosity

The magnitude of η is given by

$$\eta = \frac{F}{A \times \text{velocity gradient}}$$

η is defined as the frictional force per unit area of a liquid when it is in a region of unit velocity gradient .

Dimensions:

The dimensions of a force, F , (=mass x acceleration = mass x velocity/time) are MLT^{-2} . The dimensions of an area, A , are L^2 . The dimensions of velocity gradient

$$= \frac{\text{velocity change}}{\text{distance}} = \frac{L}{T} \div L = \frac{1}{T}$$

Now

$$\eta = \frac{F}{A \times \text{velocity gradient}}$$

$$\eta = \frac{MLT^{-2}}{L^2 \times \frac{1}{T}}$$

$$= ML^{-1}T^{-1}$$

Thus η can be expressed in **units**: $\text{kgm}^{-1}\text{sec}^{-1}$

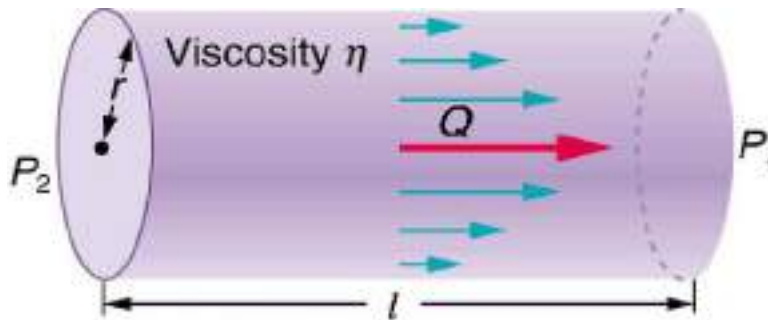
Poiseuille's Law (Flow of a fluid through a pipe)

In trying to find out what factors control how fast fluids can flow through pipes, the following factors are easy to isolate:

- The pressure difference between the ends of the pipe.
The bigger the pressure difference, the faster will be the flow.
- The length of the pipe. More liquid will flow through a shorter than a longer pipe in the same time.
- The radius of the pipe. More liquid will flow through a wide than a narrow pipe in the same time. This dependence is very marked.
- The coefficient of viscosity of the liquid. Water flows much more easily than glycerin.

Hence the volume of liquid issuing per second (Q) from the pipe depends on:

- i. The coefficient of viscosity (η).
- ii. The radius of the pipe (a).
- iii. The pressure gradient (g).



The pressure gradient = P/L , where P is the pressure difference between the ends of the pipe and L is its length.

$$Q \propto \eta^x a^y g^z$$

Thus x, y, z being indices which require to be found, suppose:

$$Q = k \eta^x a^y g^z$$

- Now the dimensions of Q (volume per second) are:

$$L^3 T^{-1}$$

- the dimensions of η are $ML^{-1}T^{-1}$

- the dimensions of g are $\frac{\text{pressure}}{\text{length}} = \frac{\text{force}}{\text{area} \times \text{length}} =$

$$\frac{MLT^{-2}}{L^3} = ML^{-2}T^{-2}$$

By equating the dimensions on both sides,

$$L^3T^{-1} \equiv (ML^{-1}T^{-1})^x L^y (ML^{-2}T^{-2})^z$$

Equating the respective indices of M, L, and T on both sides, we have

$$x + z = 0$$

$$-x + y - 2z = 3$$

$$x + 2z = 1$$

Solving we obtain $x = -1$, $z = 1$, $y = 4$. This lead to:

$$Q = k \frac{a^4 g}{\eta} = k \frac{p a^4}{L \eta}$$

The constant k was found experimentally to equal $\pi/8$

$$\boxed{\text{volume per second } (Q) = \frac{\pi p a^4}{8 L \eta}}$$

Stoke's law and terminal velocity

When a small object, such as steel ball-bearing, is dropped into a viscous liquid like glycerin it accelerates at first, but its velocity soon reaches a steady value known as the **terminal velocity**.

Suppose a sphere of radius 'r' is dropped into a viscous liquid of coefficient of viscosity 'η' and its velocity at an instant is 'v'. The frictional force 'F' can be partly found by the method of dimensions. Thus, suppose

$F = k r^x \eta^y v^z$, where k is constant. The dimensions of η are $M L^{-1} T^{-1}$ and the dimensions of v are $L T^{-1}$.

$$M L T^{-2} \equiv L^x (M L^{-1} T^{-1})^x (L T^{-1})^z$$

Equating the respective indices of M, L, and T on both sides, we have

$$y = 1,$$

$$x - y + z = 1,$$

$$-y - z = -2.$$

Hence $z = 1$, $x = 1$, $y = 1$, consequently

$$F = k r \eta v$$

Stokes showed mathematically that the constant k equal 6π , and he arrived the formula

$$F = 6\pi r \eta v$$

This equation called Stoke's law.

Determination of viscosity coefficient using Stoke's law

Suppose a tall glass vessel is filled with the liquid, and a small ball-bearing is dropped gently into the liquid so that it falls along the axis of the vessel as shown in the figure. Toward the middle of the liquid the ball reaches its terminal velocity v . In this case:

- The viscous force acting upwards ↑↑↑

$$(F = 6\pi r \eta v)$$

- The up thrust due to the liquid on the ball acting upwards ↑↑↑

$$= \rho_L V g = \rho_L \left(\frac{4}{3} \pi r^3 \right) g$$

- The weight of the ball acting downwards ↓↓↓

$$mg = \rho_s V g = \rho_s \left(\frac{4}{3} \pi r^3 \right) g$$

where, ρ_L and ρ_s are the density of liquid and ball, respectively.

At terminal velocity, the ball move with constant velocity so the two forces acting upwards equal the downward force (weight of the ball)

$$\rho_s \left(\frac{4}{3} \pi r^3 \right) g = \rho_L \left(\frac{4}{3} \pi r^3 \right) g + 6\pi r \eta v$$

$$\eta = \frac{2}{9} g (\rho_s - \rho_L) \frac{r^2}{v}$$

When this process is repeated with a liquid of coefficient of viscosity η_1 and density ρ_{L1} , using the same ball-bearing, then

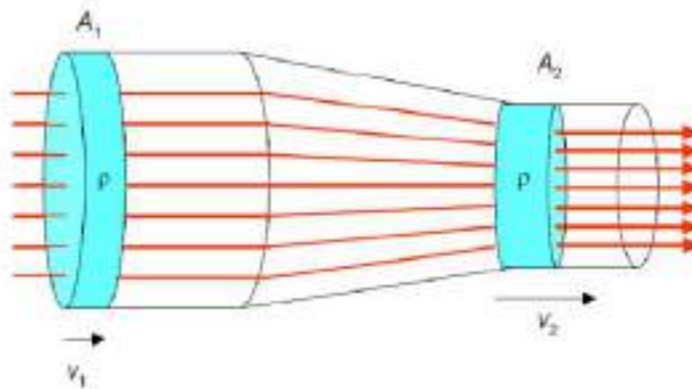
$$\eta_1 = \frac{2}{9} g (\rho_s - \rho_{L1}) \frac{r^2}{v_1}$$

where v_1 , is the new terminal velocity

$$\frac{\eta}{\eta_1} = \frac{v_1 (\rho_s - \rho_L)}{v (\rho_s - \rho_{L1})}$$

Thus knowing v , v_1 , ρ_s , ρ_L and ρ_{L1} , the coefficient of viscosity can be compared. In very accurate work a correction is required for the effect of the walls of the vessel containing the liquid.

EQUATION OF CONTINUITY



Consider an ideal fluid flowing through a pipe of varying cross sectional area A . The volume V_1 of fluid and mass m_1 flowing past (1) in a very small time interval is

$$V_1 = A_1 v_1 \Delta t$$

$$m_1 = \rho_1 A_1 v_1 \Delta t$$

Similarly the volume and mass of fluid flowing past (2) in time Δt is

$$V_2 = A_2 v_2 \Delta t$$

$$m_2 = \rho_2 A_2 v_2 \Delta t$$

When the flow is steady all the material which goes past (1) must go past (2) in the same time (or else it will continually piling up past somewhere) and since the fluid is incompressible its density does not change

$$\rho_1 = \rho_2 = \rho$$

Therefore we must have

$$m_1 = m_2$$

$$\rho A_1 v_1 \Delta t = \rho A_2 v_2 \Delta t$$

$$A_1 v_1 = A_2 v_2$$

If the fluid is approximately incompressible, i.e. if its density never changes by very much, then the equation of continuity, as we quoted it, is approximately true.

The quantity $A v$ which measures the volume of the fluid that flows past any point of the tube divided by time is called the volume flow rate $Q = dV/dt$

The equation of continuity is often expressed as

$$Q = A v = \text{constant}$$

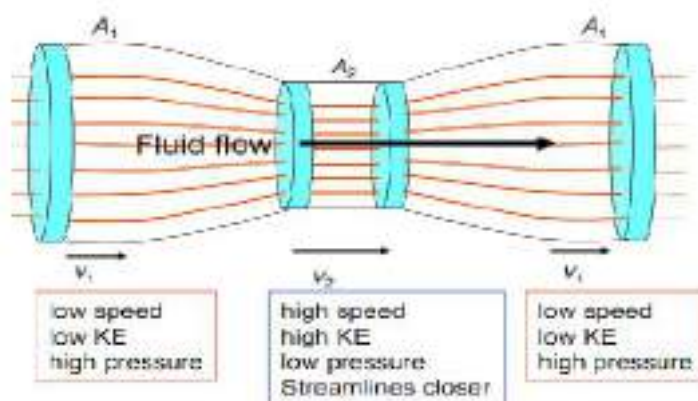
⇒ If A decreases then v increases

⇒ If A increases then v decreases

In complicated patterns of streamline flow, the streamlines effectively define flow tubes. So the equation of continuity says that where streamlines crowd together the flow speed must increase.

Bernoulli's Principle:

Bernoulli obtained a relation between the pressure and velocity at different parts of a moving incompressible fluid. If the viscosity is negligibly small, there are no frictional forces to overcome. In this case the work done by pressure forces equal the sum of both kinetic energy and the potential energy.



Derivation of Bernoulli's equation

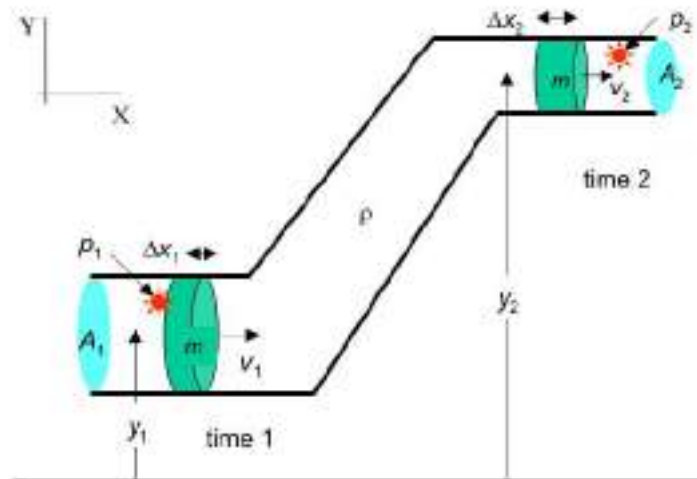
Mass element m moves from (1) to (2) as shown in the figure:

$$m = \rho A_1 \Delta X_1 = \rho A_2 \Delta X_2 = \rho \Delta V$$

where $\Delta V = A_1 \Delta X_1 = A_2 \Delta X_2$

Equation of continuity $A v = \text{constant}$

$$A_1 v_1 = A_2 v_2 \quad A_1 > A_2 \Rightarrow v_1 < v_2$$



Since $v_1 < v_2$ the mass element has been accelerated by the net force

$$F_1 - F_2 = P_1 A_1 - P_2 A_2$$

Mass element has an increase in K.E

$$\Delta K = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \frac{1}{2}\rho\Delta V v_2^2 - \frac{1}{2}\rho\Delta V v_1^2$$

Mass element has an increase in P.E

$$\Delta U = mgy_2 - mgy_1 = \rho\Delta V y_2 - \rho\Delta V y_1$$

The increase in K.E and P.E comes from the net work done on the mass element by the forces F_1 and F_2 (the sample of mass m , in moving from a region of higher pressure to a region of lower pressure, has positive work done on it by the surrounding fluid)

$$W_{net} = F_1\Delta X_1 - F_2\Delta X_2 = p_1A_1\Delta X_1 - p_2A_2\Delta X_2$$

$$W_{net} = p_1\Delta V_1 - p_2\Delta V_2 = \Delta K + \Delta U$$

Substitute

$$\begin{aligned} p_1\Delta V_1 - p_2\Delta V_2 &= \\ &= \frac{1}{2}\rho\Delta V v_2^2 - \frac{1}{2}\rho\Delta V v_1^2 + \rho\Delta V y_2 - \rho\Delta V y_1 \end{aligned}$$

Rearranging

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

This is Bernoulli's equation.

We can also write, for any point along a flow tube or streamline

$$p + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

Bernoulli's theorem state that: "the sum of pressure plus K.E per unit volume plus the P.E per unit volume always constant"

- Bernoulli's theorem only applies to an ideal fluid.
- Cannot use Bernoulli's Principle when the viscosity is significant. Can be applied to gases provided there are only small changes in pressure

Problems

1. A sphere of radius r moving through a fluid of density ρ with high velocity v experience a retarding force F given by $F = k a^x \eta^y v^z$, where k is a non-dimensional coefficient. Use the method of dimensions to find the values of x , y and z .
2. A sphere of radius 2 cm and mass 100 gm, falling vertically through air of density 1.2 kg m^{-3} , at a place where the acceleration due to gravity is 9.80 m s^{-2} , attains a steady velocity of 15 m s^{-1} . Explain why a constant velocity is reached and use the data to find the value of k in this case.
3. Two spherical rain drops of equal size are falling vertically through air with a terminal velocity of 15 ms^{-1} . What would be the terminal velocity if these two drops were to coalesce to form a larger spherical drop? (Stoke's law may be assumed to apply).